How to Discount Small Probabilities[*]

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ABSTRACT: Maximizing expected value leads to counterintuitive choices in cases that involve tiny probabilities of huge payoffs. In response to such cases, some have argued that we ought to discount very small probabilities down to zero. In this paper, I discuss how exactly this view can be formulated. I begin by showing that less plausible versions of discounting small probabilities violate dominance. Then, I show that more plausible formulations of this view avoid these dominance violations, but instead, they violate the axiom of Independence. As a result of this violation, those who discount small probabilities can be exploited by a money pump. I conclude that discounting small probabilities faces significant problems that undermine its plausibility as a theory of instrumental rationality.

Orthodox decision theory claims that a rational agent always maximizes expected utility. However, this seems to imply counterintuitive choices in cases that involve very small probabilities of huge payoffs. In these cases, an option can be great in expectation even if the probability of obtaining a valuable outcome is tiny, as long as this valuable outcome is great enough. One example of such a case is *Pascal's Mugging*:[1]

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[1]Bostrom (2009). This case is based on informal discussions by various people, including Eliezer Yudkowsky (2007*b*).

> **Pascal's Mugging:** A stranger approaches Pascal and claims to be an Operator from the Seventh Dimension. He promises to perform magic that will give Pascal an extra thousand quadrillion happy days in the Seventh Dimension if he pays the mugger ten livres—money that the mugger will use for helping very many orphans in the Seventh Dimension.

Pascal thinks that the probability of the mugger telling the truth is very low. However, the potential payoff is so high that Pascal is forced to conclude that the expected utility of paying the mugger is positive. Furthermore, if Pascal gives a non-zero probability to the proposition that the mugger can reward him with any finite amount of utility, then the mugger can always increase the payoff until the offer has positive expected utility.[2] Consequently, maximizing expected utility (with unbounded utilities) requires paying the mugger—which seems counterintuitive.[3]

Another case that involves tiny probabilities of huge payoffs is the St. Petersburg game, a version of which was originally proposed by Nicolaus Bernoulli.[4] This game is played by flipping a fair coin until it lands on heads. The prize of this game is $\$2^n$, where n is the number of coin flips. This game has infinite expected monetary value, so agents who maximize expected monetary value would pay any finite amount to play the game—but this seems counterintuitive.[5] Furthermore, if this game's (monetary) value is infinite, one would value it higher than any of its possible finite payoffs, which seems irrational.[6]

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[2]Contrary to this, see Hanson (2007), Yudkowsky (2007*a*) and Baumann (2009).

[3]This may not hold if utilities are bounded as standard axiomatizations of expected utility maximization (such as the von Neumann-Morgenstern utility theorem) require. See Kreps (1988, p. 63).

[4]The game was simplified by Gabriel Cramer in 1728 and published by Daniel Bernoulli in 1738. See Pulskamp (n.d.) and Bernoulli (1954).

[5]Pulskamp (n.d., p. 6). Daniel Bernoulli (cousin of Nicolaus Bernoulli) argues that, due to the diminishing marginal utility of money, one should not pay any finite sum to play the St. Petersburg game. See Bernoulli (1954). However, Menger (1967, pp. 217–218) shows that if utilities are unbounded, one can always create a *Super St-Petersburg game*, in which the payoffs grow sufficiently fast so that the expected utility of the game is infinite. See also Samuelson (1977, §2).

[6]Huemer (2016, pp. 34–35) and Russell and Isaacs (2021).

In response to cases like this, some have argued that we ought to discount very small probabilities down to zero—let's call this *Probability Discounting*. Nicolaus Bernoulli first proposed this idea in response to the St. Petersburg game. He writes: "[T]he cases which have a very small probability must be neglected and counted for nulls, although they can give a very great expectation. […] This is a remark which merits to be well examined."[7] Recently, Smith (2014) and Monton (2019) have also defended the idea of Probability Discounting. Monton argues that one ought to discount very small probabilities down to zero, while Smith argues that it is rationally permissible—but not required—to do so.[8] However, we do not yet have a well-specified and plausible theory that tells us how to discount small probabilities. As Monton writes: "I don't have a perfectly rational, reasonable decision theory to hand you just yet (sorry)."[9]

This paper discusses how Probability Discounting can be formulated and what the most plausible version of it might look like. §1 discusses a simple version of Probability Discounting on which one should conditionalize on outcomes associated with tiny probabilities not occurring. I show that this view faces a problem with individuating outcomes, and it also violates Statewise Dominance. §2 discusses a version of Probability Discounting that considers very-small-probability outcomes as tiebreakers when prospects would otherwise be equally good. I show that this view also violates Statewise Dominance. §3 discusses a version of Probability Discounting on which one should conditionalize on very-small-probability *states* not occurring. I discuss three ways of specifying this view. I show that one violates Stochastic Dominance and Acyclicity within choice sets, another violates Stochastic Dominance and Pairwise Acyclicity, and the last violates Statewise Dom-

[7] Pulskamp (n.d., p. 2). Other proponents of Probability Discounting include, for example, Buffon and Condorcet. See Hey et al. (2010) and Monton (2019, pp. 16–17).

[8] Smith argues that discounting small probabilities down to zero is a way of getting a unique expected value for the Pasadena game. See Nover and Hájek (2004). See Hájek (2014), Isaacs (2016) and Kosonen (2022) for criticism of discounting small probabilities. Also, see Beckstead (2013, ch. 6), Beckstead and Thomas (2020), Goodsell (2021), Russell and Isaacs (2021), Russell (2021) and Wilkinson (2022) for discussions of related issues.

[9] Monton (2019, p. 15).

inance. §4 discusses more plausible versions of Probability Discounting that avoid the earlier violations of dominance and Acyclicity. However, these views violate the axiom of Independence. As a result of this violation, those who discount small probabilities are vulnerable to exploitation by a money pump for Independence.[10] I conclude that Probability Discounting faces significant problems that undermine its plausibility as a theory of instrumental rationality.

1 Naive Discounting

This section discusses a version of Probability Discounting on which one should conditionalize on outcomes associated with tiny probabilities not occurring. I show that this view faces the *Outcome Individuation Problem*, and it also violates Statewise Dominance. Therefore, it is implausible as a theory of instrumental rationality.

According to Probability Discounting, an agent is rationally required or permitted to discount very small probabilities down to zero. On this view, there is some discounting threshold t such that probabilities below this threshold are discounted down to zero.[11] But when are probabilities small enough to be discounted? Or, as Buffon writes: "[O]ne can feel that it is a certain number of probabilities that equals the moral certainty, but what number is it?"[12] Some possible discounting thresholds have been suggested. For Buffon and Condorcet, the discounting thresholds were 1/10,000 and 1/144,768 (respectively), while for Monton, this threshold is approximately 1 in 2 quadrillion.[13] As Monton argues, the discounting

[10]Isaacs (2016) also presents a problem for Probability Discounting in a dynamic context, to which Smith (2016) and Monton (2019) respond by arguing that relevantly similar choices ought to be evaluated collectively.

[11]Alternatively, this threshold probability t and probabilities below it are discounted, while the probabilities above t are not discounted. Note that this threshold might also be vague.

[12]Hey et al. (2010, p. 256).

[13]Buffon's (Monton, 2019, pp. 8–9) discounting threshold was the probability of a 56-year-old man dying in 24 hours—an outcome reasonable people typically ignore. Condorcet (Monton, 2019, pp. 16–17) had a similar justification for his threshold. Monton's (2019, p. 17) discounting threshold is between $1/2^{50}$ and $1/2^{51}$, as he treats the probability of getting tails at least 50 times in a row

threshold is plausibly subjective. There is no objective answer to Buffon's question. Instead, it is up to each individual where the discounting threshold is.[14,15]

So, on this view, one should discount small probabilities—but small probabilities of *what*? This paper discusses versions of Probability Discounting that ignore very-small-probability outcomes or states.[16] I will begin with the former views. There are many ways of ignoring outcomes associated with small probabilities. One way to ignore the very-small-probability outcomes of some prospect $\mathcal{P}_1$ would be to treat $\mathcal{P}_1$ as interchangeable with a prospect $\mathcal{P}_2$, which really does assign probability zero to these outcomes.[17] However, $\mathcal{P}_2$ cannot assign the same probabilities as $\mathcal{P}_1$ to the remaining outcomes; otherwise, the sum of all the probabilities assigned to outcomes of $\mathcal{P}_2$ would be less than one.[18] Instead, the probabilities assigned by $\mathcal{P}_2$ can be obtained from those assigned by $\mathcal{P}_1$ by conditionalizing on the supposition that some outcome of non-negligible probability occurs, where 'non-negligible' means a probability that is at least as great as the discounting threshold.[19]

Let $X \succsim Y$ mean that X is at least as preferred as Y. Also, let $EU(X)_{pd}$ denote the expected utility of prospect X when tiny probabilities have been discounted down to zero (read as 'the probability-discounted expected utility of X'). A *prospect* is taken to be a situation that may result in different outcomes with different probabilities. One of the simplest versions of Probability Discounting—let's call it *Naive Discounting*—states:

(with a fair coin) as rationally negligible.

[14]The subjectivity of the threshold may be reasonable for individuals' rational preferences. But it seems less so in the context of ethics when we are asking which prospects are better or worse.

[15]Smith (2014) holds that the threshold might not apply to simple prospects, that is, prospects that assign a non-zero probability to only finitely many outcomes. Also, Smith maintains that this threshold may be different in different situations.

[16]Whether one ignores very-small-probability outcomes or states makes a difference in some cases. A very-small-probability state might result in some outcome that overall has a non-negligible probability (when one also considers the other states). In that case, the state is associated with a negligible probability but the outcome is not.

[17]Smith (2014, p. 478).

[18]Smith (2014, p. 478).

[19]Smith (2014, p. 478).

> **Naive Discounting:** For all prospects X and Y, $X \succsim Y$ if and only if $EU(X)_{pd} \geq EU(Y)_{pd}$, where $EU(X)_{pd}$ and $EU(Y)_{pd}$ are obtained by conditionalizing on the supposition that some outcome of non-negligible probability occurs.[20]

Outcome Individuation Problem. So, on Naive Discounting, one should conditionalize on very-small-probability outcomes not occurring—but what counts as an 'outcome'? In particular, Naive Discounting faces the following problem:[21]

> **Outcome Individuation Problem:** If we individuate outcomes with too much detail, all outcomes have negligible probabilities. Is there a privileged way of individuating outcomes that avoids this?

The most obvious non-arbitrary way of individuating outcomes is by their utilities:[22]

> **Individuation by Preference:** Outcomes should be distinguished as different if and only if one has a preference between them.

Following this principle, each final utility level that a prospect might result in is considered a distinct outcome, and the possibilities of these outcomes are ignored if their associated probabilities are below the discounting threshold.

However, individuating outcomes by their utilities might result in ignoring all possible outcomes of some prospect if all its final utility levels are very unlikely. In response to such cases, agents might lower their discounting thresholds until at

[20]Note that $EU(X)_{pd}$ and $EU(Y)_{pd}$ are obtained by conditionalizing, potentially, on different events not occurring.

[21]See also Beckstead and Thomas (2020, p. 13).

[22]Contrast Individuation by Preference with a similar principle presented by Broome (1991, p. 103):

> **Principle of Individuation by Justifiers:** Outcomes should be distinguished as different if and only if they differ in a way that makes it rational to have a preference between them.

least some outcomes have non-negligible probabilities. However, in cases where all outcomes have a zero probability, it is not possible to do so (except, of course, by not discounting at all).[23] Imagine, for example, an ideally shaped dart thrown on a dartboard, where each point results in a different utility. The probability that the dart hits a particular point may be zero. But one should not ignore every possible outcome of throwing the dart. Nevertheless, one might argue that we need not worry about cases where all outcomes have a zero probability because they are rare in practice. In all (or near all) cases we care about, some outcomes have non-zero probabilities.

Statewise Dominance. Some might be satisfied with the above solution to the Outcome Individuation Problem. However, besides this problem, Naive Discounting also violates dominance. Let $X \succ Y$ mean that X is strictly preferred (or simply 'preferred') to Y. Then, Naive Discounting violates the following dominance principle:[24]

> **Statewise Dominance:** If the outcome of prospect X is at least as preferred as the outcome of prospect Y in all states, then $X \succsim Y$. Furthermore, if in addition the outcome of X is strictly preferred to the outcome of Y in some possible state, then $X \succ Y$.

Statewise Dominance is very plausible. If some prospect is sure to turn out at least as well as another prospect, but it might turn out better, then that prospect should be better.[25]

To see why Naive Discounting violates Statewise Dominance, consider the following prospects (see table 1):[26]

[23] Beckstead and Thomas (2020, pp. 12–13).

[24] Savage (1951, p. 58) and Luce and Raiffa (1957, p. 287).

[25] Russell (2021, p. 13) writes on (strict) Statewise Dominance: "What if Statewise Dominance fails? In that case, I'm not sure what we're doing when we compare how good prospects are. […] [W]hat we ultimately care about is how well things turn out; choosing better prospects is supposed to guide us toward achieving better outcomes. In light of this, if dominance reasoning is wrong, then I don't want to be right. If A is sure to turn out better than B, then this tells us precisely the thing that betterness-of-prospects is supposed to be a guide to."

[26] On discounting small probabilities and dominance violations, see Isaacs (2016), Smith (2016),

Naive Statewise Dominance Violation:

Prospect A Gives $1,000,000 in state 1 and nothing in state 2.

Prospect B Gives nothing in both states.

Suppose the probability of state 1 is below the discounting threshold. After conditionalizing on the supposition that some outcome of non-negligible probability occurs, A is substituted by B. One would then be indifferent between A and B, even though the outcomes of A and B are equally good in state 2, but the outcome of A is better than the outcome of B in state 1.

Table 1
Naive Statewise Dominance Violation

	State 1 $p <$ threshold	**State 2** $1 - p$
A	$1,000,000	$0
B	$0	$0

To summarize, Naive Discounting states that one should conditionalize on not obtaining very-small-probability outcomes. This view faces the Outcome Individuation Problem, which can be solved by individuating outcomes by their utilities (except in cases where all outcomes have a zero probability). However, Naive Discounting also faces another problem: It violates Statewise Dominance. This undermines its plausibility as a theory of instrumental rationality.[27]

Monton (2019, pp. 20–21), Lundgren and Stefánsson (2020, pp. 912–914) and Beckstead and Thomas (2020, §2.3).

[27] Hájek (2014) shows that Expected Utility Theory also violates Statewise Dominance in cases that involve possible states of zero probability. Monton (2019, §7) argues that violations of Statewise Dominance should not count against Probability Discounting, given that Expected Utility Theory violates Statewise Dominance too. Later in §4, I discuss versions of Probability Discounting that do not violate Statewise Dominance.

2 Lexical Discounting

This section discusses a version of Probability Discounting that treats very-small-probability outcomes as tiebreakers when prospects would otherwise be equally good. This view avoids the previous violation of Statewise Dominance. However, I show that it violates Statewise Dominance in another case.

There is a straightforward solution to the previous case: Treat outcomes whose probabilities are below the discounting threshold as tiebreakers. Then, A is better than B because A and B have equal probability-discounted expected utility but, in addition, A gives a negligible probability of a positive outcome (while B does not). More generally, in tied cases, prospects can be compared by their expected utilities without any discounting (like Expected Utility Theory would do).

On this proposal, prospects are first ranked by their probability-discounted expected utilities. Then, in cases of ties, these prospects are ranked by their expected utilities without discounting small probabilities. Formally this view—let's call it *Lexical Discounting*—states the following:

> **Lexical Discounting:** For all prospects X and Y, $X \succsim Y$ if and only if
>
> - $EU(X)_{pd} > EU(Y)_{pd}$ or
> - $EU(X)_{pd} = EU(Y)_{pd}$ and $EU(X) \geq EU(Y)$,
>
> where $EU(X)_{pd}$ and $EU(Y)_{pd}$ are obtained by conditionalizing on the supposition that some outcome of non-negligible probability occurs.

It is slightly misleading to say that Lexical Discounting is a form of discounting small probabilities down to zero because small probabilities and their associated utilities are considered in cases of ties. The outcomes whose probabilities are (at and) above the discounting threshold just take lexical priority over the very-small-probability outcomes.[28]

[28]It might be argued that because some small probabilities are much smaller than others, one

Statewise Dominance. Lexical Discounting also violates Statewise Dominance. Consider the following case (table 2):

Lexical Statewise Dominance Violation:

Prospect A Gives $10 in states 1 and 2, $100 in state 3, and nothing in state 4.

Prospect B Gives $10 in state 1, $100 in states 2 and 3, and nothing in state 4.

The probability of states 1 and 4 is 0.49, and the probability of states 2 and 3 is 0.01. For simplicity, let the discounting threshold be (implausibly) 0.03. Let's also assume that the utility of money equals the monetary amount.

Table 2

Lexical Statewise Dominance Violation

	State 1	**State 2**	**State 3**	**State 4**
p	0.49	0.01	0.01	0.49
A	$10	$10	$100	$0
B	$10	$100	$100	$0

After conditionalizing on not obtaining $100 with A (as its associated probability is below the discounting threshold), A's probability-discounted expected utility is $EU(A)_{pd} \approx 5.05$.[29] And B's probability-discounted expected utility is $EU(B)_{pd} = 5$ after conditionalizing on not obtaining $100 with it.[30] Given that the former is greater than the latter, A is better than B according to Lexical Discounting. However, the only difference between A and B is that A gives $10 in

should have multiple discounting thresholds that form probability ranges, where higher probability ranges take lexical priority over the lower ones.

[29] $0.5/(1 - 0.01) \cdot 10 \approx 5.05$.

[30] $0.49/(1 - 0.02) \cdot 10 = 5$.

state 2, while B gives \$100 in that same state. Therefore, Lexical Discounting—too—violates Statewise Dominance.

This violation of Statewise Dominance happens because when one conditionalizes on not obtaining \$100 with A (state 3), the probability of state 3 is divided between states 1, 2 and 4. However, when one conditionalizes on not obtaining \$100 with B (states 2 and 3), the probability of states 2 and 3 is divided between states 1 and 4. Therefore, the probability of obtaining nothing is greater with B than with A after ignoring the possibility of obtaining \$100.

To summarize, Lexical Discounting states that outcomes whose probabilities are (at or) above the discounting threshold take lexical priority over very-small-probability outcomes in determining prospects' betterness ranking—very-small-probability outcomes are only treated as tiebreakers. However, like Naive Discounting, Lexical Discounting also violates Statewise Dominance. This makes it a less plausible candidate for a theory of instrumental rationality.

3 State Discounting

This section discusses a version of Probability Discounting on which one should conditionalize on very-small-probability states not occurring. Three versions of this view are presented. I show that one violates Stochastic Dominance and Acyclicity within choice sets, another violates Stochastic Dominance and Pairwise Acyclicity, and the last one violates Statewise Dominance.

3.1 Pairwise and Set-Dependent State Discounting

Again, there is a straightforward solution to the previous violation of Statewise Dominance. Earlier it was assumed that one should ignore (except in cases of ties) the possibility of obtaining outcomes associated with tiny probabilities. However, one might instead ignore very-small-probability *states*—call this view *State Discounting*. One can also make a lexical version of this view:

State Discounting For all prospects X and Y, $X \succsim Y$ if and only if

- $EU(X)_{pd} > EU(Y)_{pd}$ or
- $EU(X)_{pd} = EU(Y)_{pd}$ and $EU(X) \geq EU(Y)$,

where $EU(X)_{pd}$ and $EU(Y)_{pd}$ are obtained by conditionalizing on the supposition that no state of negligible probability occurs.

In the previous violation of Statewise Dominance, State Discounting tells one to ignore states 2 and 3 as their associated probabilities are below the discounting threshold. Consequently, A and B have equal probability-discounted expected utility (as they give the same outcomes in states 1 and 4). However, B has greater expected utility without discounting, so it is better than A (assuming a lexical version of State Discounting). Thus, State Discounting avoids the previous violation of Statewise Dominance.[31] However, next I will present some problems for State Discounting.

State Individuation Problem. State Discounting faces an analogous problem to the Outcome Individuation Problem, namely, the

> **State Individuation Problem:** If one individuates states with too much detail, all states have negligible probabilities. Is there a privileged way of individuating states that avoids this?

As before, a possible solution is to individuate states by the utilities of their outcomes.[32]

However, there are different views about how to exactly this should be done. On one version of State Discounting, prospects are always compared two at a time, and the possible states of the world are partitioned for every pairwise comparison separately. Alternatively, one could compare all available options at once and

[31] However, as I will show later, one version of State Discounting violates Statewise Dominance in this case.

[32] As before, one problem with this is that, in some cases, all states might have probabilities below the discounting threshold. One could lower the threshold in such cases. However, this will not solve the problem in cases where all states have a zero probability.

partition the states for every choice set separately. Let's call these views *Pairwise State Discounting* and *Set-Dependent State Discounting*, respectively (the following Acyclicity violation illustrates the difference between these views).

> **Pairwise State Discounting:** States are partitioned by comparing two prospects at a time.

> **Set-Dependent State Discounting:** States are partitioned by comparing all available prospects at once.

ACYCLICITY. Although both versions of State Discounting avoid the earlier violations of Statewise Dominance (in the way explained before), they violate the following principle instead:

> **Acyclicity:** If $X_1 \succ X_2 \succ \cdots \succ X_n$, then it is not the case that $X_n \succ X_1$.

To see why these views violate Acyclicity, consider the following case:

> **Acyclicity Violation:** A random number generator returns a number between 1 and 100.
>
> *Prospect* A Gives \$1000 with numbers 1 and 2 (probability 0.02); otherwise it gives nothing.
>
> *Prospect* B Certainly gives \$10 no matter what number comes up.
>
> *Prospect* C Gives \$1000 with number 1 (probability 0.01) and otherwise it gives \$1.

Let the discounting threshold be 0.02. First, compare A and B. Individuating states by the utilities of their outcomes results in two states as shown in table 3. A is better than B because neither state has a non-negligible probability, and A's expected utility is greater than that of B.[33] Next, compare B and C. In this case, individuating states by the utilities of their outcomes results in states shown in table 4. As

[33] $EU(A)_{pd} = 0.02 \cdot 1000 = 20$ and $EU(B)_{pd} = 10$.

the probability of state 1* is below the discounting threshold, one should ignore the possibility of state 1* occurring. Once one does that, B is better than C, as it gives a better outcome in state 2* ($10 vs. $1).

Table 3
A is better than B

Output	State 1 1 or 2 (p=0.02)	State 2 3 to 100 (p=0.98)
A	$1000	$0
B	$10	$10

Table 4
B is better than C

Output	State 1* 1 (p=0.01)	State 2* 2 to 100 (p=0.99)
B	$10	$10
C	$1000	$1

Now we have that A is better than B, which is better than C. It follows by Acyclicity that C is not better than A. However, when we compare A and C pairwise, we notice that C is better than A. In this case, individuating states by the utilities of their outcomes results in states shown in table 5. As states 1** and 2** have probabilities below the discounting threshold, the agent should ignore the possibilities of these states. Then, C is better than A because it gives a better outcome in state 3**. So, we have a violation of Acyclicity: A is better than B, which is better than C, which is better than A.

Table 5
C is better than A

Output	State 1** 1 (p=0.01)	State 2** 2 (p=0.01)	State 3** 3 to 100 (p=0.98)
A	$1000	$1000	$0
C	$1000	$1	$1

Let's now go back to Pairwise and Set-Dependent State Discounting. If we partition states for each pair of options in a way that depends on the particular two options being compared (in line with Pairwise State Discounting), then State Discounting violates Acyclicity within choice sets. Consequently, it is not clear what one ought to choose when all A, B and C are available, as there is no most-preferred alternative.[34]

However, if we partition states in a way that depends on the overall choice set (in line with Set-Dependent State Discounting), then there is no violation of Acyclicity within choice sets (see table 6). In this case, states 1*** and 2*** have probabilities below the discounting threshold, so one should ignore the possibilities of these states. Consequently, B is the best prospect as it gives the best outcome in state 3***, and C is the second-best prospect as it gives a better outcome than A in that state.

Table 6

No Violation of Acyclicity

Output	**State 1****** 1 (p=0.01)	**State 2****** 2 (p=0.01)	**State 3****** 3 to 100 (p=0.98)
A	\$1000	\$1000	\$0
B	\$10	\$10	\$10
C	\$1000	\$1	\$1

However, Set-Dependent State Discounting violates Acyclicity across choice sets (as shown in tables 3, 4 and 5). In particular, it was shown that Set-Dependent State Discounting violates Pairwise Acyclicity, that is, it violates Acyclicity when we compare two options at a time.

It is odd that adding or removing options can influence which events one ignores. For example, when comparing A and B, Set-Dependent State Discounting does not ignore the possibility of the random number generator outputting number 1 or 2. However, when C is also available, Set-Dependent State Discounting ignores

[34]Fishburn (1991, p. 116).

these possibilities. Consequently, the value of A decreases significantly when C is also available, as one then ignores the possibility of obtaining \$1000 with A.[35]

Stochastic Dominance. Finally, I will show that both versions of State Discounting violate the following principle:[36]

Stochastic Dominance: Prospect $X = \{x_1,\ p_1; \ldots ; x_n,\ p_n\}$ is preferred to prospect $Y = \{y_1,\ q_1; \ldots ;\ y_n,\ q_n\}$ if, for all outcomes o,

$$\sum_{\{i \mid x_i \succsim o\}} p_i \geq \sum_{\{j \mid y_j \succsim o\}} q_j,$$

and for some outcome u,

$$\sum_{\{i \mid x_i \succsim u\}} p_i > \sum_{\{j \mid y_j \succsim u\}} q_j.$$

A violation of Stochastic Dominance happens if, for all outcomes, some prospect X gives an at least as high probability of an at least as great outcome as some other prospect Y does, and for some outcome, X gives a greater probability of an at least as great outcome as Y does—yet Y is judged better than or equally as good as X.

To see why both versions of State Discounting violate Stochastic Dominance, consider the following case:

Two Coins:

Prospect A Gives \$10 if a coin lands on heads (p = 0.5), nothing if it lands on tails (p = 0.49), and \$100 if it lands on the edge (p = 0.01).

[35] This case shows that Set-Dependent State Discounting violates *Contraction Consistency* and *Strong Expansion Consistency.* See Sen (1977, pp. 63–66). More generally, Contraction Consistency implies Acyclicity. See Sen (1977, p. 67).

[36] Buchak (2013, p. 42). More precisely, the definition given here is for *first-order stochastic dominance*, an idea that was introduced to statistics by Mann and Whitney (1947) and Lehmann (1955), and to economics by Quirk and Saposnik (1962). The name 'first-degree stochastic dominance' is due to Hadar and Russell (1969, p. 27).

Prospect B Gives \$10 if another coin lands on heads (p = 0.49), nothing if it lands on tails (p = 0.49), and \$100 if it lands on the edge (p = 0.02).

Let the discounting threshold be 0.03. These prospects give the same probabilities of the same outcomes as the prospects in *Lexical Statewise Dominance Violation* (table 2). But instead of four states, we now have nine different states due to having two coins. Let 'H' stand for 'heads', 'T' for 'tails' and 'E' for 'edge'. Also, let '(X, Y)' stand for the first coin landing on 'X' and the second one on 'Y'. Only four states have probabilities above the discounting threshold: (H, H), (H, T), (T, H) and (T, T) (see table 7).

Table 7

Two Coins

	H, H	**H, T**	**T, H**	**T, T**
p^*	*0.253*	*0.253*	*0.247*	*0.247*
A	\$10	\$10	\$0	\$0
B	\$10	\$0	\$10	\$0

p^*=probability conditional on one of these states occurring.

After conditionalizing on one of these four states occurring, the probability-discounted expected utility of A is greater than that of B. Now the only difference between these prospects is that A gives \$10 in state (H, T) and nothing in state (T, H), while B gives \$10 in the latter state and nothing in the former one—and the former state has a greater probability.[37] Thus, A is better than B according to both versions of State Discounting. However, this is a violation of Stochastic Dominance. Before discounting, both A and B give a 0.51 probability of at least \$10, but B gives a greater probability of at least \$100 (0.02 vs. 0.01). So, for all outcomes, B gives an at least as high probability of an at least as great outcome as A does, and for some outcome, B gives a greater probability of an at least as

[37] $EU(A)_{pd} \approx 5.05$ and $EU(B)_{pd} = 5$.

great outcome as A does. Thus, B stochastically dominates A—and both versions of State Discounting violate Stochastic Dominance.

3.2 Baseline State Discounting

According to the previous versions of State Discounting, states might be partitioned differently depending on what other options are available. This leads to a violation of Acyclicity. However, states might also be partitioned in a way that does not depend on the other available options. This can be done by comparing each prospect to some baseline or status quo prospect—let's call this *Baseline State Discounting*.

> **Baseline State Discounting:** States are partitioned by comparing every prospect to a status quo prospect (each separately).[38]

Statewise Dominance. However, Baseline State Discounting violates Statewise Dominance in the same way as Lexical Discounting does. Consider again *Lexical Statewise Dominance Violation* (table 2). This time, let's specify the events that result in each outcome:

> **Random Number:** A random number generator returns a number between 1 and 100.
>
> *Prospect A* Gives \$10 with numbers 1 to 50, \$100 with number 51 and nothing with numbers 52 to 100.
>
> *Prospect B* Gives \$10 with numbers 1 to 49, \$100 with numbers 50 and 51 and nothing with numbers 52 to 100.

In this case, the baseline prospect is (presumably) certainly getting nothing. When A is compared to this baseline prospect, state individuation by utilities results in

[38]Note that, on Baseline State Discounting, one might sometimes ignore some events e_1 and e_2 when comparing some prospect X to the status quo prospect, but not ignore them when comparing another prospect Y to the status quo prospect.

three states as shown in table 8. As the probability of state 2 is below the discounting threshold of 0.03, the possibility of this state is ignored. Consequently, the probability-discounted expected utility of A is $EU(A)_{pd} \approx 5.05$.[39]

Table 8

A vs. the Baseline

Output	**State 1** 1–50 (p=0.5)	**State 2** 51 (p=0.01)	**State 3** 52–100 (p=0.49)
A	\$10	\$100	\$0
Baseline	\$0	\$0	\$0

Next, compare B to the baseline prospect. This time state individuation by utilities results in the three states shown in table 9. Again, the probability of state 2* is below the discounting threshold, so the possibility of this state is ignored. Then, the probability-discounted expected utility of B is $EU(B)_{pd} = 5$.[40]

Table 9

B vs. the Baseline

Output	**State 1*** 1–49 (p=0.49)	**State 2*** 50 or 51 (p=0.02)	**State 3*** 52–100 (p=0.49)
B	\$10	\$100	\$0
Baseline	\$0	\$0	\$0

As A's probability-discounted expected utility is greater than that of B (5.05 vs. 5), A is better than B. However, B statewise dominates A because the only difference between these prospects is that A gives \$10 if the random number generator returns the number 50, while B gives \$100 in that case. Consequently, Baseline State Discounting violates Statewise Dominance when states are partitioned

[39] $EU(A)_{pd} = 0.5/0.99 \cdot 10 \approx 5.05$.

[40] $EU(B)_{pd} = 0.49/0.98 \cdot 10 = 5$.

in the usual way corresponding to possible states of the world (such as 'number 50 is returned'). This violation of Statewise Dominance happens because the possible states of the world that Baseline State Discounting ignores are not the same for every prospect. For example, when comparing A to the baseline prospect, the possibility of the random number generator returning number 50 is not ignored, but this possibility is ignored when B is compared to the baseline prospect.

To summarize, instead of ignoring very-small-probability outcomes, Probability Discounting might ignore very-small-probability states. State Discounting faces the State Individuation Problem, which can be solved by individuating states by the utilities of their outcomes. I have discussed three ways of formulating State Discounting. Pairwise State Discounting always compares two options at a time and ignores very-small-probability states in every pairwise comparison. However, Pairwise State Discounting violates Stochastic Dominance and Acyclicity within choice sets. Set-Dependent State Discounting compares all available options simultaneously and ignores very-small-probability states in every choice set. This view violates Pairwise Acyclicity and Stochastic Dominance. Finally, Baseline State Discounting ignores very-small-probability states of some prospect when states are partitioned by comparing this prospect to a baseline prospect. This view violates Statewise (and hence also Stochastic) Dominance. To conclude, all three versions of State Discounting violate plausible principles of rationality.[41]

4 Stochastic and Tail Discounting

This section discusses more plausible versions of Probability Discounting that avoid the earlier violations of dominance (and Acyclicity). However, like all versions of Probability Discounting, these views violate the axiom of Independence and are

[41] Someone might adopt a view on which one should first filter one's options by Statewise and Stochastic Dominance and then choose following some version of Probability Discounting from amongst the remaining options. This view avoids the dominance violations, but it also seems *ad hoc*. However, some may find the benefit of a greater fit with our intuitions worth the cost in terms of simplicity.

therefore vulnerable to exploitation by a money pump.

4.1 Stochastic Discounting

One version of Probability Discounting—let's call it *Absolutist Stochastic Discounting*—works like this: To obtain the probability-discounted expected utility of a prospect, first add the lowest possible (positive) utility, weighted by the probability of getting at least that much utility. Next, add the difference between the lowest utility and the next lowest utility, weighted by the probability of getting at least the higher amount of utility. Then, add the difference between this utility and the next lowest utility, weighted by the probability of getting at least that much utility, and so on until the next probability is below the discounting threshold.[42] Then, ignore the rest of the utility levels (whose probabilities are below the discounting threshold). Negative utilities are then treated similarly, and their expectation is summed with the expectation of positive utilities to obtain the value of a prospect.

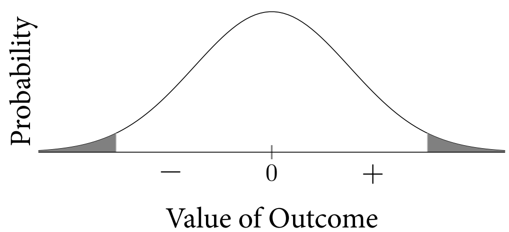
According to Absolutist Stochastic Discounting, there is an objective neutral level. On this view, one should ignore the possibility of very high or very low utility levels when the probability of ending up with at least or at most that much utility (respectively for positive and negative utilities) is negligible. This view recommends against paying the mugger in Pascal's Mugging if there is only a tiny probability that one will get an outcome at least as good as a thousand quadrillion happy days. However, it does not recommend against paying the mugger if there is a non-negligible probability of obtaining an outcome that is at least as great as a thousand quadrillion happy days for some reason unrelated to the mugger's offer.[43]

Consider a prospect that has possible outcomes whose values are normally dis-

[42]This is similar to an alternative way of calculating the expected utility of a prospect discussed by Buchak (2014, p. 1100).

[43]For example, an agent who thinks there is a non-negligible probability of going to Heaven would not ignore the possibility of a great payoff in Pascal's Mugging. More generally, such an agent would not discount small probabilities very often (if ever); the non-negligible probability of going to Heaven makes it the case that there is a non-negligible probability of ending up with at least u amount of utility for all positive values of u.

tributed with a mean of zero when the outcomes are ordered in terms of betterness. Absolutist Stochastic Discounting tells one to ignore the highest positive and the lowest negative utility levels of this prospect, that is, to ignore the utility levels of the outcomes in the grey areas (see the graph below).



Call the versions of Probability Discounting that have the same structure as Absolutist Stochastic Discounting *Stochastic Discounting*. There is another way of understanding Stochastic Discounting. This view is similar to Baseline State Discounting, as it compares each prospect to a baseline prospect—call it *Baseline Stochastic Discounting*. On this view, one calculates the amount by which the baseline/status quo utility level is increased or decreased by the different possible outcomes of a prospect. Then, to obtain the probability-discounted expected utility of a prospect, one first adds the lowest possible gain (i.e., positive change to the baseline), weighted by the probability of gaining at least that much. Next, one adds the difference between the lowest gain and the next lowest gain, weighted by the probability of gaining at least the higher amount, and so on until the next probability is below the discounting threshold. Then, one ignores the rest of the possible gains. Losses (i.e., negative changes to the baseline) are treated similarly, and their expectation is summed with the expectation of gains to obtain the value of a prospect.[44]

Absolutist Stochastic Discounting has the (possible) disadvantage of requiring an objective neutral utility level. Baseline Stochastic Discounting does not require

[44]One can also make a version of Stochastic Discounting that is analogous to Pairwise State Discounting in that it compares prospects to other available prospects pairwise—call this *Pairwise Stochastic Discounting*. On this view, one considers the utility difference in each state between two prospects and ignores the largest differences when the cumulative probability of states with differences at least that large is negligible.

one because it ignores very large changes to the baseline—the baseline serves the same purpose as the objective neutral level on the absolutist view.

Also, Absolutist Stochastic Discounting implies that sometimes one might not ignore the possibility of a huge gain (or loss) even if there is only a tiny probability of it occurring. This can happen if one would end up with a negative outcome regardless of the gain, and the probability of obtaining an outcome that is at most as good as that is non-negligible. Baseline Stochastic Discounting, in contrast, ignores a tiny probability of a great gain even in that case.

Similarly, on the absolutist view one should not ignore a tiny probability of a huge gain if the probability of ending up with a higher utility level is non-negligible for some reason unrelated to the prospect in question. As mentioned before, this view does not recommend against paying the mugger in Pascal's Mugging if there is a non-negligible probability of gaining a greater payoff for some reason unrelated to the mugger's offer. Baseline Stochastic Discounting, in contrast, recommends against paying the mugger even in that case. This is so because once one has 'subtracted' the baseline prospect from the mugger's offer, gains at least as great as a thousand quadrillion happy days have a negligible cumulative probability. So, unlike Baseline Stochastic Discounting, Absolutist Stochastic Discounting sometimes lets tiny probabilities of huge gains or losses dictate one's course of action. Therefore, it does not capture the motivation behind Probability Discounting as well as Baseline Stochastic Discounting does.

On both versions of Stochastic Discounting, the probability-discounted expected utility of positive outcomes is calculated as follows (here 'positive outcomes' are either gains if one accepts Baseline Stochastic Discounting or final utilities if one accepts Absolutist Stochastic Discounting):[45]

[45]Technically, this formula requires the following qualifications: If all probabilities of positive utility levels are non-negligible, then in order to obtain $EU(X)_{pd,\ pos}$, one simply sums up the positive utilities weighted by their probabilities (without discounting). And if all probabilities of positive utility levels are negligible, then $EU(X)_{pd,\ pos} = 0$ (on Baseline Stochastic Discounting) or the value of the baseline (on Absolutist Stochastic Discounting). Furthermore, this formula assumes that, amongst the possible positive utility levels, there is one that is the lowest.

Positive outcomes: For all prospects X, such that X gives non-zero probabilities of positive outcomes
$X_{pos} = \{E_1,\ x_1;\ E_2,\ x_2; ...;\ E_m,\ x_m; ...;\ E_n,\ x_n\}$, and
$0 < u(x_1) \leq ... \leq u(x_m) \leq ... \leq u(x_n)$, the probability-discounted expected utility of positive outcomes of X is

$$\begin{aligned} EU(X)_{pd,\ pos} =& \Big(\sum_{i=1}^{n} p(E_i)\Big)u(x_1) + \Big(\sum_{i=2}^{n} p(E_i)\Big)(u(x_2) - u(x_1)) \\ &+ \Big(\sum_{i=3}^{n} p(E_i)\Big)(u(x_3) - u(x_2)) \\ &+ \cdots + \Big(\sum_{i=m}^{n} p(E_i)\Big)(u(x_m) - u(x_{m-1})), \end{aligned}$$

where

$$\sum_{i=m}^{n} p(E_i) \geq t > \sum_{i=m+1}^{n} p(E_i),$$

where t is the discounting threshold.[46]

The probability-discounted expected utility of prospect X is obtained by summing the probability-discounted expected utilities of its positive and negative outcomes.[47] And, as before, Stochastic Discounting can use very-small-probability utility levels as tiebreakers to rank prospects with equal probability-discounted expected utility. It can then be stated as follows:

[46] The following formula is equivalent to the formula given above:

$$EU(X)_{pd,\ pos} = \sum_{i=1}^{m} p(E_i)u(x_i) + \Big(\sum_{i=m+1}^{n} p(E_i)\Big)u(x_m),$$

where x_m is the greatest positive utility that has a non-negligible cumulative probability, and x_n is the greatest positive utility possible with prospect X.

[47] The probability-discounted expected utility of negative outcomes (i.e. $EU(X)_{pd,\ neg}$) is calculated in a similar way (changing what needs to be changed) as that of positive outcomes.

Stochastic Discounting: For all prospects X and Y, $X \succsim Y$ if and only if

- $EU(X)_{pd} > EU(Y)_{pd}$ or
- $EU(X)_{pd} = EU(Y)_{pd}$ and $EU(X) \geq EU(Y)$,

where, for all prospects X, it holds that

$$EU(X)_{pd} = EU(X)_{pd,\ pos} + EU(X)_{pd,\ neg}.$$

Now, recall the earlier violations of Statewise and Stochastic Dominance (*Lexical Statewise Dominance Violation*, *Random Number* and *Two Coins*). Unlike the earlier versions of Probability Discounting, both versions of Stochastic Discounting imply that B is better than A. Both A and B give a 0.51 probability of at least \$10. In addition, A gives a 0.01 probability of at least \$100, while B gives a 0.02 probability of at least \$100. After ignoring the possibility of obtaining \$100, the probability-discounted expected utility of A and B is $EU(A)_{pd} = EU(B)_{pd} = 5.1$.[48] As A and B have equal probability-discounted expected utility, these prospects are then compared by their expected utilities without discounting. Consequently, B is better than A, and both versions of Stochastic Discounting avoid the earlier violations of Statewise and Stochastic Dominance.[49]

4.2 Tail Discounting

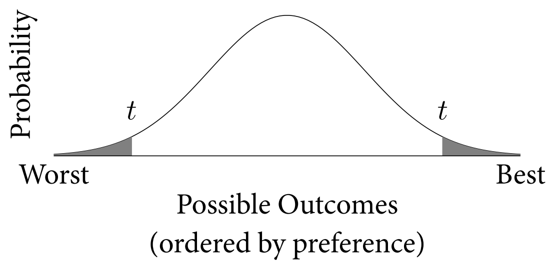
There is a similar view to Absolutist Stochastic Discounting called *Tail Discounting*.[50] According to Tail Discounting, one should ignore both the left and the right

[48] $EU(A)_{pd} = EU(B)_{pd} = 0.51 \cdot 10 = 5.1$.

[49] Wilkinson (2022, §6) shows that views that reject Probability Fanaticism must violate separability or Stochastic Dominance. Absolutist Stochastic Discounting violates the former. Given that Baseline Stochastic Discounting ignores background uncertainty (and thus satisfies separability), it must sometimes violate Stochastic Dominance.

[50] Tail Discounting is from Beckstead and Thomas (2020, §2.3).

'tails' of the distribution of possible outcomes of some prospect X when these outcomes are ordered by one's preference. Suppose the possible outcomes of some prospect are normally distributed when they are ordered from the least to the most preferred. Then, Tail Discounting advises one to ignore the grey areas under the curve:



Call the outcomes that fall in the middle of the distribution of possible outcomes 'normal outcomes'. An outcome is normal if and only if there is a non-negligible probability of getting at least and at most as good an outcome. Tail Discounting then states the following:

Tail Discounting: For all prospects X and Y, $X \succsim Y$ if and only if

- $EU(X)_{pd} > EU(Y)_{pd}$ or
- $EU(X)_{pd} = EU(Y)_{pd}$ and $EU(X) \geq EU(Y)$,

where $EU(X)_{pd}$ and $EU(Y)_{pd}$ are obtained by conditionalizing on the supposition that some normal outcome occurs.

Tail Discounting has the advantage over Absolutist Stochastic Discounting that it does not require an objective neutral level. However, similarly to Absolutist Stochastic Discounting, Tail Discounting recommends paying the mugger in Pascal's Mugging if, for some reason unrelated to the mugger's offer, there is a non-negligible probability of obtaining a greater outcome. This is because then a thousand quadrillion happy days falls in the middle part of the distribution of possible outcomes, which is not ignored.[51]

Again, recall the earlier violations of Statewise and Stochastic Dominance (*Lexical Statewise Dominance Violation*, *Random Number* and *Two Coins*). Tail Discounting also implies that B is better than A. Again, A and B have equal probability-discounted expected utility: $EU(A)_{pd} = EU(B)_{pd} \approx 5.1$.[52] These prospects are then compared by their expected utilities without discounting. Thus, B is better than A—and Tail Discounting avoids the earlier violations of Statewise and Stochastic Dominance.

To summarize, I have discussed three versions of Probability Discounting in this section. Absolutist Stochastic Discounting states that one should ignore the possibility of a very high (very low) utility level in cases where the probability of obtaining at least (at most) that much utility is below the discounting threshold. Baseline Stochastic Discounting works similarly, but it operates on gains and losses instead of final utilities. Lastly, Tail Discounting states that one should ignore the 'tails' of the distribution of possible outcomes of some prospect. All these views avoid the earlier violations of Statewise and Stochastic Dominance (and Acyclicity). However, next, I will raise a diachronic problem for these views.

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[51] One might also make a version of Tail Discounting similar to Baseline Stochastic Discounting. On this view—let's call it *Baseline Tail Discounting*—one compares every prospect to a baseline prospect as follows: First, calculate the difference in utilities a prospect makes in each state (compared to the baseline prospect). Then, order these differences from the greatest loss to the greatest gain. Next, ignore the right and left tails of this distribution by conditionalization. Also, one can make a version of Tail Discounting similar to Pairwise Stochastic Discounting (i.e., *Pairwise Tail Discounting*). On this view, one compares prospects pairwise instead of comparing every prospect to a baseline prospect.

[52] $EU(A)_{pd} = (0.5-0.02)/0.94 \cdot 10 \approx 5.1$ and $EU(B)_{pd} = (0.49-0.01)/0.94 \cdot 10 \approx 5.1$. The divisor '0.94' comes from subtracting the discounting threshold of 0.03 from both tails of the distribution. '0.02' and '0.01' are subtracted to make sure that the full discounting threshold of 0.03 is ignored in the right tail. In general, on Tail Discounting, one discounts a little bit of each 'tail' with every prospect (until the discounting threshold is ignored from both tails).

4.3 Independence violation

This section shows that Stochastic and Tail Discounting violate the axiom of Independence. As a result of this violation, these views are vulnerable to exploitation by a money pump.

Let XpY be a risky prospect with a p chance of prospect X obtaining and a $1 - p$ chance of prospect Y obtaining. Then, Independence states the following:

> **Independence:** If $X \succ Y$, then $XpZ \succ YpZ$ for all probabilities $p \in (0, 1]$.[53]

Informally, Independence is the idea that every outcome contributes to the value of a prospect in a way that does not depend on the alternative outcomes.

The basic problem for Probability Discounting is that by mixing gambles, one can arbitrarily reduce the probabilities of different states or outcomes within the compound lottery until these probabilities end up below the discounting threshold. Therefore, mixtures of gambles can end up being valued differently than the gambles that are mixed together. For example, consider the following case:

> **Independence Violation:**
>
> *Prospect A* Certainly gives nothing.
>
> *Prospect B* Gives a 0.5 probability of \$1 and otherwise −\$1,000,000.
>
> *Prospect C* Certainly gives \$1.

Next, let $p = 0.02$. Then, we have the following mixed prospects (see table 10):

> *Prospect ApC* Gives a 0.98 probability of \$1 and otherwise nothing.
>
> *Prospect BpC* Gives a 0.99 probability of \$1 and a 0.01 probability of −\$1,000,000.

[53] Jensen (1967, p. 173).

Table 10

Independence Violation

p	0.01	0.01	0.98
ApC	\$0	\$0	\$1
BpC	−\$1,000,000	\$1	\$1

First, consider what Stochastic Discounting says about these prospects (Baseline and Absolutist Stochastic Discounting treat this case similarly if the agent possesses nothing when making this choice). Let the discounting threshold be 0.02. ApC gives a 0.98 probability of gaining at least \$1 (and otherwise nothing), so its probability-discounted expected utility is $EU(ApC)_{pd} = 0.98$. BpC, in turn, gives a 0.99 probability of gaining at least \$1 and a 0.01 probability of losing at least \$1,000,000. The probability of losing at least \$1,000,000 is below the discounting threshold, so this possibility is ignored. Thus, BpC's probability-discounted expected utility is $EU(BpC)_{pd} = 0.99$. So, according to Stochastic Discounting, BpC is better than ApC.

Next, consider what Tail Discounting says about these prospects. Now let the discounting threshold be 0.01. Then, Tail Discounting also implies that BpC is better than ApC. After ignoring both tails of the distribution of possible outcomes of ApC, its probability-discounted expected utility is $EU(ApC)_{pd} \approx 0.99$.[54] And after ignoring the tails of the distribution of possible outcomes of BpC, its probability-discounted expected utility is $EU(BpC)_{pd} = 1$.[55] Thus, we have that BpC is better than ApC.

Some might consider this implication already worrisome on its own, but it is also a violation of Independence. Both Stochastic and Tail Discounting consider A better than B. It is better to get nothing certainly than to take a 50–50 gamble between gaining \$1 and losing \$1,000,000. Thus, we have the following violation of Independence:

[54] $(0.98 - 0.01)/0.98 \cdot 1 \approx 0.99$.

[55] $(0.99 - 0.01)/0.98 \cdot 1 = 1$.

$A \succ B$, and $BpC \succ ApC$ for some probability $p \in (0, 1]$.

This renders Stochastic and Tail Discounting vulnerable to exploitation by a money pump for Independence.[56] A money-pump argument intends to show that agents who violate some alleged requirement of rationality are vulnerable to making a combination of choices that leads to a sure loss. If vulnerability to this kind of exploitation is a sign of irrationality, then Stochastic and Tail Discounting are untenable as theories of instrumental rationality. Moreover, this violation of Independence is particularly counterintuitive because BpC is considered better than ApC no matter how bad the negative outcome (−$1,000,000) is as long as the good outcome ($1) is at least slightly positive.

To summarize, Stochastic and Tail Discounting violate Independence, which renders those who accept these views vulnerable to exploitation by a money pump. This makes these views—and Probability Discounting more generally—less plausible as theories of instrumental rationality. But what should one do now? One could, for example, bite the bullet and accept a version of Probability Discounting discussed in this paper, find a more plausible version of Probability Discounting, bound utilities[57], conditionalize on one's knowledge before maximizing expected utility[58] or accept probability fanaticism.[59]

5 Conclusion

Expected utility maximization with unbounded utilities implies counterintuitive choices in cases that involve tiny probabilities of huge payoffs. In response to

[56] See Hammond (1988*b*, pp. 292–293), Hammond (1988*a*, pp. 43–45), Gustafsson (2021, p. 31 n21) and Gustafsson (2022, §5).

[57] See Beckstead and Thomas (2020) and Kosonen (2022).

[58] See for example Francis and Kosonen (n.d.).

[59] See for example Beckstead and Thomas (2020) and Wilkinson (2022). However, note that, independently of Probability Discounting, agents with unbounded utilities are also vulnerable to money pumps because they violate countable generalizations of the Independence axiom. See Russell and Isaacs (2021).

such cases, some have argued that we should discount small probabilities down to zero. I have discussed how exactly this view can be formulated. First, I argued that less plausible versions of Probability Discounting violate dominance. More specifically, I showed that Naive Discounting, Lexical Discounting and Baseline State Discounting violate Statewise Dominance. I also showed that Pairwise State Discounting violates Stochastic Dominance and Acyclicity within choice sets and that Set-Dependent State Discounting violates Pairwise Acyclicity and Stochastic Dominance.

Then, I showed that more plausible versions of Probability Discounting, namely Stochastic Discounting and Tail Discounting, avoid these dominance violations. However, they violate the axiom of Independence. As a result of this violation, those who accept these views can be exploited by a money pump. This makes them—and Probability Discounting more generally—less plausible as theories of instrumental rationality. All in all, I have discussed possible ways of formulating Probability Discounting. All of these theories have significant problems, and it is yet to be seen whether there is a perfectly rational, reasonable decision theory that deviates from Expected Utility Theory by discounting small probabilities down to zero.

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