Chapter 1

Bounded Utilities and Ex Ante Pareto[*]

ABSTRACT: This chapter shows that decision theories on which utilities are bounded, such as standard axiomatizations of Expected Utility Theory, violate Ex Ante Pareto if combined with an additive axiology, such as Total Utilitarianism. A series of impossibility theorems point toward Total Utilitarianism as the right account of axiology, while money-pump arguments put Expected Utility Theory in a favorable light. However, it is not clear how these two views can be reconciled. This question is particularly puzzling if utilities are bounded (as standard axiomatizations of Expected Utility Theory imply) because the total quantity of well-being might be infinite or arbitrarily large. Thus, there must be a non-linear transformation from the total quantity of well-being into utilities used in decision-making. This non-linear transformation is also required if one wishes to avoid Probability Fanaticism. However, such a transformation leads to violations of Ex Ante Pareto. So, the reconciliation of Expected Utility Theory and Total Utilitarianism prescribes prospects that are better for none and worse for some.

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This chapter investigates the compatibility of two standard theories: Total Utilitarianism and Expected Utility Theory with a bounded utility function. Let's call the combination of these views *Bounded Expected Totalism*. Unfortunately, this chapter argues that Bounded Expected Totalism violates *Ex Ante Pareto*, the principle that what is ex ante better for everyone is better overall.[1,2] This principle is often used by utilitarians to justify their theory in opposition to others, such as prioritarianism and egalitarianism.

Insofar as Expected Utility Theory is the dominant theory of choice under uncertainty, this argument could be seen as undermining Total Utilitarianism. However, I ultimately take the lesson to be different. As I explained in the introductory chapter, considerations of Probability Fanaticism motivate either the use of a bounded utility function or some form of Probability Discounting (or perhaps some third option). So, I take the arguments in this chapter to speak differentially in favor of Probability Discounting. Actually, as I will explain in Chapter 3, Probability Discounting also leads to violations of Ex Ante Pareto. So, to put the point more accurately, the plausibility of Ex Ante Pareto does not favor Bounded Expected Utility Theory over Probability Discounting.

The chapter proceeds as follows. I will first define Bounded Expected Total-

[1] There is no inconsistency with Harsanyi's social aggregation theorem. As will be explained later, a bounded expected totalist must reject Harsanyi's conclusion, so they cannot accept all his premises.

[2] This chapter focuses on the compatibility of Expected Utility Theory and Total Utilitarianism, but the problem with Ex Ante Pareto arises for, for example, Critical-Level Utilitarianism in exactly the same way. The problem also arises for Average Utilitarianism and many other theories if individual utilities are unbounded. See for example the argument in Goodsell (2021), which applies to any axiology that is utilitarian in same-number cases. This chapter shows that, even if utilities are bounded, Total Utilitarianism combined with Expected Utility Theory violates Ex Ante Pareto.

ism more formally and explain why this is a prima facie attractive view. I will then proceed to illustrate why this view must violate Ex Ante Pareto. A background issue, which is laid out in §2.2, is how the well-being of a single individual can be traded off between different states of nature. The question is essentially whether the personal value of prospects is risk-averse with respect to well-being. I will give separate examples of Ex Ante Pareto violations that involve risk-neutrality (§3) and risk-aversion (§4). §5.2 gives a general argument for why Bounded Expected Totalism must violate Ex Ante Pareto. I conclude in §6 by sketching how my examples relate to the classic result in this area, namely, Harsanyi's social aggregation theorem.

1 Background

This section introduces some background. First, it explains Total Utilitarianism and Expected Utility Theory. Then, it discusses the idea that utilities are bounded and why this follows from standard axiomatizations of Expected Utility Theory. Lastly, it discusses bounded utilities as a possible way of getting intuitively right recommendations in cases that involve tiny probabilities of huge payoffs.

1.1 Total Utilitarianism and Expected Utility Theory

A series of impossibility theorems point toward Total Utilitarianism as the right account of axiology, while money-pump arguments put Expected Utility Theory in a favorable light.[3] The former view states that a population is better than another just in case the total quantity of well-being it contains is greater, while the latter states that a prospect is better than another just in case its expected utility is greater.[4] Let $X \succsim Y$ mean that X is at least as good as Y. Also, let $\mathrm{W}(A)$ denote the total quantity of well-being in the state of affairs A and let $\mathrm{w}(S_i)$ denote the well-being of individual S_i. Then, more formally, Total Utilitarianism states the following:

> **Total Utilitarianism:** For all states of affairs A and B (in which n and m individuals exist, respectively), $A \succsim B$ if and only if $\mathrm{W}(A) \geq \mathrm{W}(B)$, where
>
> $$\mathrm{W}(A) = \sum_{i=1}^{n} w(S_i) \text{ and } \mathrm{W}(B) = \sum_{i=1}^{m} w(S_i).^{5}$$

Next, let $\mathrm{EU}(X)$ denote the expected utility of prospect X, $p(E_i)$ the probability of event E_i and $u(x_i)$ the utility of outcome x_i (which results from event E_i). Then, Expected Utility Theory states the following:[6]

[3] See for example Arrhenius (2000) and Gustafsson (forthcoming). The impossibility theorems point toward Total Utilitarianism because they show that we cannot escape the Repugnant Conclusion without being forced to accept even more unpalatable conclusions. See also Zuber et al. (2021).

[4] In the case of Total Utilitarianism, 'better' is used in an axiological sense; in the case of Expected Utility Theory, 'better' is concerned with instrumental rationality.

[5] To cover cases in which an infinite number of individuals exist in state of affairs A, we may extend Total Utilitarianism as follows:

$$\mathrm{W}(A) = \sum_{i=1}^{\infty} w(S_i).$$

[6] I am assuming that prospects can be countably infinite, that is, assign a non-zero probability

Expected Utility Theory: For all prospects X and Y, $X \succsim Y$ if and only if $\mathrm{EU}\left(X\right) \geq \mathrm{EU}\left(Y\right)$, where

$$\mathrm{EU}\left(X\right) = \sum_{i=1}^{\infty} p\left(E_i\right) u\left(x_i\right).$$

Combining Total Utilitarianism and Expected Utility Theory with a bounded utility function, we get *Bounded Expected Totalism*:

Bounded Expected Totalism: Total Utilitarianism and Expected Utility Theory with a bounded utility function are both true.

1.2 Boundedness

What does it mean for utilities to be bounded? If utilities are real-valued, then boundedness means the following:

Boundedness: There is some $M \in \mathbb{R}$ such that for all outcomes x, $|u(x)| < M$.

In other words, Boundedness rules out arbitrarily and infinitely good outcomes.

Standard axiomatizations of expected utility maximization require utilities to be bounded.[7] Consider, for example, the von Neumann-Morgenstern axiomatization of Expected Utility Theory.[8] Let $X \succ Y$ mean that X is better than Y. Also, let

to countably infinite number of outcomes. This assumption is needed because some of the cases discussed in this chapter involve such prospects.

[7] See for example Kreps (1988, pp. 63–64), Fishburn (1970, pp. 194, 206–207), Hammond (1998, pp. 186–191) and Russell & Isaacs (2021).

[8] The following axioms together entail Expected Utility Theory: Completeness, Transitivity, In-

XpYbe a risky prospect with a p chance of prospect X obtaining and a $1-p$ chance of prospect Yobtaining. Then, if prospects are compared by their expected utilities, Boundedness follows from the following von Neumann-Morgenstern axiom:

> **Continuity:** If $X \succ Y \succ Z$, then there are probabilities p and $q \in (0,1)$ such that $XpZ \succ Y \succ XqZ$.

To see why Continuity implies Boundedness (assuming that prospects are compared for their expected utilities), let's consider the two ways in which Boundedness might be false.[9] First, Boundedness might be false because there is an infinite sequence of prospects $A_1, A_2, A_3, \ldots$ such that A_2 is at least twice as good as A_1, A_3 is at least twice as good as A_2, and so on, with respect to some baseline. Let A be a mixed prospect that assigns probability $1/2^k$ to prospect A_k. Then, we have that

$$\mathrm{EU}\left(A\right) = \sum_{i=1}^{\infty} p\left(A_i\right) u\left(A_i\right) = \infty.$$

dependence and Continuity. See von Neumann & Morgenstern (1947), Jensen (1967, pp. 172–182) and Hammond (1998, pp. 152–164).

[9] Boundedness is false if *Limitedness* or *Finiteness* is false:

> **Limitedness:** There is no infinite sequence of prospects $X_1, X_2, X_3, \ldots$ such that X_2 is at least twice as good (bad) as X_1, X_3 is at least twice as good (bad) as X_2, and so on, with respect to some baseline Z.
>
> **Finiteness:** No prospect is infinitely better (worse) than another good (bad) prospect.

Limitedness is from Russell & Isaacs (2021, p. 12). Limitedness, unlike Finiteness, allows infinite utilities (as long as there are no series of at least twice as good prospects with infinite expected utility). Russell & Isaacs (2021) show that Countable Independence rules out violations of Limitedness (via St. Petersburg-style cases). However, Countable Independence does not rule out unbounded utilities, as some prospects might still be infinitely better than other prospects. Given St. Petersburg-style cases, Finiteness implies Limitedness.

Next, choose some prospects B and C such that $\infty > \text{EU}(B) > \text{EU}(C) > -\infty$. Then, we have that A is better than B, which is better than C. However, for all probabilities $q \in (0, 1)$, $\text{EU}(AqC) = \infty$. Therefore, AqC is better than B for all probabilities $q \in (0, 1)$. This is a violation of Continuity.[10]

Secondly, and more generally, Boundedness is false if some prospect A is infinitely better than another (good) prospect B. This leads to a violation of Continuity because the mixed prospect ApC (where C certainly gives nothing) is better than B for all probabilities $p \in (0, 1)$. So, the supposition that Boundedness is false leads to violations of Continuity. Thus, it follows from Continuity that Boundedness is true.[11]

1.3 Probability Fanaticism

Boundedness has been discussed as a possible alternative to Probability Fanaticism.[12] Probability Fanaticism is the idea that tiny probabilities of large positive or negative payoffs can have enormous positive or negative expected utility (respectively):[13]

Probability Fanaticism:

i *Positive Probability Fanaticism* For any probability $p > 0$, and

[10] This is a modified argument from Kreps (1988, pp. 63–64).

[11] These arguments show that Continuity implies an upper bound on utilities. One can give similar arguments to show that Continuity implies a lower bound on utilities.

[12] See for example Beckstead & Thomas (2020).

[13] Wilkinson (2022, p. 449). For discussions related to Probability Fanaticism, see Beckstead (2013, ch. 6), Beckstead & Thomas (2020), Goodsell (2021), Russell & Isaacs (2021), Russell (2021) and Wilkinson (2022).

for any finite utility u, there is some large enough utility U such that probability p of U (and otherwise nothing) is better than certainty of u.[14]

ii *Negative Probability Fanaticism* For any probability $p > 0$, and for any finite negative utility $-u$, there is some large enough negative utility $-U$ such that probability p of $-U$ (and otherwise nothing) is worse than certainty of $-u$.

If utilities are bounded, then sufficiently small probabilities of even very good (or very bad) outcomes do not contribute much to the expected utility of a prospect. For a given probability, there is an upper/lower bound on the contribution to expected utility from outcomes associated with that probability. If the probability gets smaller, this bound also shrinks proportionally so that small enough probabilities cannot help but contribute only a small amount of expected (positive or negative) utility.

For any tiny probability of a great outcome, there is still some certain modest positive outcome that is worse. However, it is not the case that for any certain modest positive outcome, an *arbitrarily* small probability of a sufficiently great outcome is better. If the probability of the great outcome is small enough, increases in the payoff can no longer compensate for decreases in its probability. So, Boundedness prevents such outcomes from dominating the expected utility calculations, and thus, it escapes Probability Fanaticism (assuming fixed upper and lower bounds

[14] In this context, 'otherwise nothing' means retaining the status quo or baseline outcome.

on utilities).

Let's call a case *fanatical* if tiny probabilities of enormous positive or negative outcomes dominate the expected utility calculations in that case. One example of a fanatical case is *Pascal's Mugging*:[15]

> **Pascal's Mugging:** A stranger approaches Pascal and claims to be an Operator from the Seventh Dimension. The stranger promises to perform magic that will help quadrillions of orphans in the Seventh Dimension if Pascal pays the mugger ten livres.

Pascal thinks that the mugger is almost certainly lying. However, if utilities are unbounded, the mugger can always increase the payoff until the offer has positive expected utility—at least if Pascal assigns some non-zero probability to the mugger being able and willing to deliver any finite quantity of utility for Pascal.[16] Then, with some number of orphans, the expected-utility-maximizing act is to pay the mugger ten livres. Moreover, the mugger can also ask for more money and increase the payoff accordingly. So, someone who maximizes expected utility with an unbounded utility function would be willing to pay any sum, provided that the payoff is sufficiently large.

In contrast, Bounded Expected Totalism has upper and lower bounds on utilities. Consequently, there is an upper limit to how much a bounded expected to-

[15] Bostrom (2009). The case presented here is a slightly modified version of Bostrom's case. In Bostrom's case, the mugger promises to give Pascal an extra thousand quadrillion happy days and help many orphans in the Seventh Dimension. The case is based on informal discussions by various people, including Eliezer Yudkowsky (2007*b*).

[16] Contrary to this, see Hanson (2007), Yudkowsky (2007*a*) and Baumann (2009).

talist would be willing to pay the mugger (assuming fixed upper and lower bounds on utilities). Bounded Expected Totalism does not escape the mugging entirely because, for any payoff offered by the mugger, there is *some* amount a bounded expected totalist would pay. After all, a tiny chance of obtaining the upper or avoiding the lower limit of utilities is worth something. But at least a bounded expected totalist would not lose all their money.[17] So, Bounded Expected Totalism helps avoid the worst instances of Probability Fanaticism (again assuming fixed upper and lower bounds on utilities).

However, this chapter shows that, under some circumstances, Bounded Expected Totalism violates Ex Ante Pareto: It prescribes prospects that are better for no one and worse for some.

> **Ex Ante Pareto:** For all prospects X and Y, if X is at least as good as Y for everyone, and X is better than Y for some, then X is better than Y.

Bounded Expected Totalism violates Ex Ante Pareto if there is a non-zero probability that an infinite or arbitrarily large number of individuals exist. But it also violates Ex Ante Pareto if it avoids Probability Fanaticism (as I will explain shortly).

2 Bounded Expected Totalism

This section presents Bounded Expected Totalism in more detail and discusses the cardinal structure of well-being.

[17] This may not be true if the mugger repeatedly returns with the same offer.

2.1 The social transformation function

Let *well-being* refer to how good some outcome is for an individual. And, let *social utility* refer to how good some outcome is overall, from an axiological point of view. Also, let *expected individual utility* represent how good some prospect is for an individual, and let *expected social utility* represent how good some prospect is overall. In the context of Expected Utility Theory, I will denote these by $\mathrm{EU}_{\mathrm{Ind}}$ and $\mathrm{EU}_{\mathrm{Soc}}$, respectively. In general, I will use *individual betterness* to refer to betterness from an individual's point of view. Similarly, I will use *overall/impersonal betterness* to refer to betterness from a moral point of view.

To combine Total Utilitarianism and Expected Utility Theory, we need a *social transformation function* that takes the total quantity of well-being as input and gives social utilities as output. This transformation function must be non-linear if an infinite or arbitrarily large number of happy individuals might exist, as then the total sum of individuals' well-being might be infinite or arbitrarily large (and similarly for negative well-being).[18] But Bounded Expected Totalism requires expected social utilities to be bounded. So, the expected social utilities assigned to prospects that might result in an infinite or arbitrarily large number of happy individuals must be bounded.[19]

[18] Note that the total quantity of well-being is not necessarily infinite if an infinite number of individuals exist. For example, suppose that for each individual $k \in \{1, 2, \ldots\}$, k's well-being measure takes a value in the interval $(0, 2^{-k})$. Then, an infinite number of individuals exist but the total quantity of well-being is bounded. However, this can be ruled out by requiring the individual well-being measures to have the same range.

[19] Beckstead & Thomas (2020, p. 9) write that Boundedness conflicts with the most natural understanding of utilitarianism as an evaluative theory on which improving n lives by a given amount improves the world by n times as much as improving one life. Similarly, they point out that Total

One might object that the total quantity of well-being cannot be infinite or arbitrarily large because there is an upper limit to how many individuals might exist. This upper limit might be due to, for example, the Universe being finite. However, this may not be true, so we need a decision theory that can also handle these possibilities.[20] If there is even a tiny probability that an infinite or arbitrarily large number of individuals exist, then the transformation function must be non-linear for utilities to be bounded. Consider for example the following versions of Pascal's Mugging:

> **Pascal's Mugging (infinite orphans):** The mugger promises to perform magic that will help an *infinite* number of orphans in the Seventh Dimension if Pascal pays the mugger ten livres.

> **Pascal's Mugging (St. Petersburg case):** The mugger promises to perform magic that gives a $1/2$ probability of helping two orphans, a $1/4$ probability of helping four orphans, a $1/8$ probability of helping eight orphans, and so on if Pascal pays the mugger ten livres.

Suppose Pascal has a non-zero credence in the mugger telling the truth. In that case, he needs to assign some expected social utility to the possibility of helping an infinite or arbitrarily large number of orphans. And, if utilities are bounded,

Utilitarianism and its variants put unbounded value on creating good lives.

[20] As Branwen (2009) put it: "Scientists have suggested infinite universes on multiple occasions, and we cannot rule the idea out on any logical ground. Should our theory of rationality stand or fall on what the cosmologists currently think?" Also, Bostrom (2011, p. 10) writes that recent cosmological evidence suggests that the world is probably infinite, which means that it contains an infinite number of galaxies, stars and planets. And, Bostrom writes, if there are an infinite number of planets, then there is, with probability one, an infinite number of people.

then the utility assigned cannot be infinite. Thus, the social transformation function must be non-linear. Moreover, anyone could be confronted with these kind of offers. So, we need a theory that can handle cases such as these.

In the previous two cases, the mugger promises to help an infinite number of orphans in expectation, which forces the social transformation function to be non-linear.[21] However, even if the mugger does not promise to help an infinite number of individuals in expectation, Bounded Expected Totalism does not avoid Probability Fanaticism if the social transformation function is linear and there is no upper limit to how many individuals might exist. For example, the mugger can always promise to help a greater number of orphans and thus increase the payoff arbitrarily high:

> **Pascal's Mugging (any number of orphans):** The mugger promises to perform magic that will help n number of orphans, where n is finite but arbitrarily large.

If social utilities are linear with the total quantity of well-being, then Bounded Expected Totalism recommends paying the mugger any sum of money, provided that the number of orphans is sufficiently high. That is, for any tiny probability p of the mugger telling the truth, and for any sum of money x, there is some finite number of orphans n, such that Pascal ought to pay the mugger x if the mugger promises to help n orphans. Thus, Bounded Expected Totalism does not avoid

[21] We might object that Total Utilitarianism is not intended to apply in infinite cases. After all, in infinite cases, the total quantity of well-being is not well-defined. So, we might think that Total Utilitarianism does not make sense if there might be an infinite number of individuals.

Probability Fanaticism if there is no upper limit to how many individuals might exist and the social transformation function is linear.

Lastly, even if we were certain that there is an upper limit to how many individuals might exist, the total quantity of well-being might still be very large. In that case, Bounded Expected Totalism could do with a linear social transformation function, as the requirement for utilities to be bounded would already be satisfied. However, if Bounded Expected Totalism is to avoid fanatical prescriptions in cases that involve tiny probabilities of huge payoffs, then the upper and lower bounds cannot be very high or very low (respectively). So, if a very large number of individuals exist, then the transformation function must be non-linear—or Bounded Expected Totalism does not avoid Probability Fanaticism in an intuitively adequate way.

Bounded Expected Totalism would, technically, avoid Probability Fanaticism if there is an upper limit to how many individuals might exist (and individual utilities are bounded). This is because then it would not be true that, for any certain modest outcome, an arbitrarily small probability of a sufficiently great outcome is better (and similarly for negative outcomes). However, Bounded Expected Totalism would still prescribe what might be considered fanatical choices in cases that involve tiny probabilities of huge outcomes, even if there is an upper limit to how many individuals might exist. This happens because the values of those outcomes can be very high (or very low) and, thus, dominate the expected utility calculations. For example, Bounded Expected Totalism might advise Pascal to pay a too high a price to the mugger.

So, there are three reasons to adopt a non-linear social transformation function: First, in expectation, an infinite number of individuals might exist, and these possibilities must be assigned a bounded expected social utility. Secondly, arbitrarily many individuals might still exist, in which case Bounded Expected Totalism does not avoid Probability Fanaticism if the social transformation function is linear. Lastly, even if there is an upper limit to how many individuals might exist, the number of possible individuals might still be very large. In that case, Bounded Expected Totalism would prescribe fanatical choices in cases that involve tiny probabilities of huge outcomes.

Suppose that the social transformation function is non-linear. It will also have the following qualities: First, more well-being is always better, so the social transformation function must be strictly increasing with the total quantity of well-being; it must assign greater utilities to outcomes that contain more well-being. Secondly, because utilities are bounded above, similar increases in well-being must (after some point at least) matter less and less. Consequently, the social transformation function must be strictly concave on some subset of its domain. Furthermore, because utilities are also bounded below, similar increases in negative well-being must (after some point at least) matter less and less. Thus, the social transformation function must be strictly convex on some subset of its domain. Lastly, for utilities to be bounded, the social transformation function must be sufficiently concave with positive total well-being and sufficiently convex with negative total well-being; the contribution of additional (positive or negative) well-being to social utility must tend to zero.

Let f be this transformation function. Also, let $p(E_i)$ denote the probability of event E_i, $\mathrm{W}(A_i)$ the total quantity of well-being in state of affairs A_i (which results from event E_i) and $\mathrm{w}(S_{ij})$ the well-being of individual S_j in state of affairs A_i. Then, we can state Bounded Expected Totalism formally as follows:[22]

> **Bounded Expected Totalism:** For all prospects X and Y, $X \succsim Y$ if and only if $\mathrm{EU}_{\mathrm{Soc}}(X) \geq \mathrm{EU}_{\mathrm{Soc}}(Y)$, where
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> $$\mathrm{EU}_{\mathrm{Soc}}(X) = \sum_{i=1}^{n} p(E_i) f(\mathrm{W}(A_i)) = \sum_{i=1}^{n} p(E_i) f\left(\sum_{j=1}^{m} w(S_{ij})\right).$$

Bounded Expected Totalism is the view that outcomes are ranked by their total quantity of well-being, and prospects are ranked by expected social utility, where social utility is some bounded function of the total quantity of well-being. On Bounded Expected Totalism, when calculating the value of a prospect, one first

[22] This chapter discusses what might be called *Ex-Post Bounded Expected Totalism*. However, there is another way Bounded Expected Totalism can deal with risk. This view—let's call it *Ex-Ante Bounded Expected Totalism*—first calculates the total quantity of well-being in every possible state of the world. Then, it multiplies the total quantity of well-being of each state with the probability of that state and sums these up. Finally, it transforms the expected well-being of a prospect into its expected social utility. Formally, *Ex-Ante* Bounded Expected Totalism states the following:

> ***Ex-Ante* Bounded Expected Totalism:** For all prospects X and Y, $X \succsim Y$ if and only if $\mathrm{EU}_{\mathrm{Soc}}(X) \geq \mathrm{EU}_{\mathrm{Soc}}(Y)$, where
>
> $$\mathrm{EU}_{\mathrm{Soc}}(X) = f\left(\sum_{i=1}^{n} p(E_i)\mathrm{W}(A_i)\right) = f\left(\sum_{i=1}^{m} p(E_i) w(S_i)\right).$$

Ex-Ante Bounded Expected Totalism violates Continuity. For example, let A be a St. Petersburg-style lottery (with the outcomes being total quantities of well-being), B a prospect that certainly gives a modest good outcome and C a prospect that certainly gives nothing. The expected total well-being of the mixed prospect ApC is infinite for all $p \in (0, 1)$. Thus, the expected social utility of ApC equals the upper bound of utilities, which is greater than the expected social utility of B. So, A is better than B, which is better than C, but ApC is better than B for all $p \in (0, 1)$—which is a violation of Continuity.

calculates the total quantity of well-being in every possible state of the world. Then, one transforms each state's total quantity of well-being into social utilities. Finally, to get the expected social utility of a prospect, one multiplies the social utility of each state with that state's probability and sums these up.

2.2 The cardinal structure of well-being

As mentioned above, the social transformation function takes the total quantity of well-being as input. To make sense of 'total quantity of well-being', we need well-being to have a 'cardinal structure', which allows us to make statements about *how much* more well-being an individual has in some outcome compared to another outcome.[23]

Where does this structure come from? There are two ways of deriving the cardinal structure of well-being. First, the cardinal structure of well-being might be understood in a 'primitivist' sense, according to which it can be defined independently of the individual betterness relation on gambles.[24] Alternatively, the cardinal structure of well-being might be understood in a technical sense as, for example, von Neumann-Morgenstern utilities. On the technical understanding, if the individual betterness relation satisfies a set of axioms, it can be represented by an expectational utility function.

Broome suggests that the meaning of our quantitative notion of good (i.e., well-

[23] Note that in order to talk of 'negative utilities', a cardinal structure is not sufficient; for that, well-being must have a ratio structure—which the von Neumann-Morgenstern axioms cannot deliver. Total Utilitarianism requires a meaningful zero level of well-being, which a merely interval/cardinal scale does not provide.

[24] Greaves (2015).

being) must be determined in this way. He proposes that 'utility' embodies the results of weighing good across states of nature.[25] Broome (1991, p. 147) writes: "To say that two differences in good are the same may mean nothing more than that they count the same when weighed against each other; they are evenly balanced in determining overall good. This would mean that two differences in good are the same whenever the corresponding differences in utility are the same. And that would be enough to ensure that utility is an increasing linear transform of good. Utility, then, would measure good cardinally. [. . .] In brief, the suggestion is that our metric of good may be determined by weighing across states of nature."[26]

If von Neumann-Morgenstern utilities represent the cardinal structure of well-being, then individual betterness is, by definition, risk-neutral with respect to well-being. It might still be risk-averse with respect to money or happy years of life. But it cannot be risk-averse with respect to well-being because well-being just is the quantity whose expectation the betterness relation can be represented as maximizing. This view satisfies the following principle:[27]

> **Bernoulli's hypothesis:** One alternative is at least as good for a person as another if and only if it gives the person at least as great an

[25] Broome (1991, p. 146). Note that we need not equate utility with how much the agent values those gambles (i.e., their preferences). Utilities tell us which gambles are better and worse for a person relative to a given probability assignment, and—especially since the probability assignment at issue need not be the agent's own—this need not coincide with what the agent prefers.

[26] Broome (1991, p. 148) also concedes that we might find a metric of well-being in some other way. For example, instead of weighing up across the dimension of states of nature, he writes that this metric might be found by weighing up across a different dimension, such as the dimension of time.

[27] Broome (1991, p. 142). I have replaced 'good' with 'well-being'.

expectation of their well-being.

Bernoulli's hypothesis implies risk-neutrality about well-being.[28] It also tells us that utility represents well-being cardinally.

This chapter focuses mostly on lifetime well-being. But many of the same issues arise when we aggregate intrapersonal well-being over time.[29] Let *momentary well-being* mean how good things are for a person at some time. At least in theory, an agent can live infinitely or arbitrarily long at a given level of bliss. Therefore, for well-being/utilities to be bounded, momentary well-being must have diminishing marginal well-being/utility. Additional happy years of life must contribute less the more happy years the agent already has (and similarly for unhappy years of life).

If Bernoulli's hypothesis is false, then individual betterness might be risk-averse with respect to well-being. For example, agents might be represented as maximizing risk-weighted expected utility.[30] Alternatively, well-being could be understood

[28] Broome (1991, pp. 124 and 203).

[29] See Broome (1991, p. 226) on the *Intertemporal Addition Theorem*:

> **Intertemporal Addition Theorem:** If a person's overall betterness relation and their momentary betterness relations obey the axioms of Expected Utility Theory, and the overall betterness relation satisfies a temporal version of Pareto, then the person's overall betterness relation can be represented by an expectational utility function that is the sum of expectational utility functions representing their momentary betterness relations.

The temporal version of Pareto says that if two alternatives are equally good for a person at every time, they are equally good for them. And, if one alternative is at least as good as another for the person at every time and definitely better for them at some time, it is better for them. See Broome (1991, p. 225). The Intertemporal Addition Theorem is a variation of Harsanyi's social aggregation theorem discussed in §6 of this chapter.

[30] See for example Quiggin (1982), Buchak (2013) and Buchak (2017). Risk-weighted expected utility theory (a member of rank-dependent theories) does not avoid fanatical prescriptions in the prudential case unless individual utilities are bounded. Similarly, (impersonal) risk-weighted expected utility theory does not avoid Probability Fanaticism unless social utilities are bounded. See

in a primitivist sense. The primitivist view requires that quantities of well-being have meaning independently of how much they count when evaluating uncertain prospects.[31] But if such a metric of well-being is available, then individual betterness might be risk-averse with respect to this (non-technical) well-being. Note that this view is compatible with Expected Utility Theory (but not with Bernoulli's hypothesis).

Let an *agent's transformation function* be a function that takes that person's well-being levels as input and outputs their individual utilities (to be used in decision-making under risk). If individual betterness over prospects is sufficiently risk-averse with respect to well-being, such that the agent's transformation function approaches asymptotically some upper bound with more well-being, then well-being itself can be unbounded without leading to unbounded utilities.

Finally, individual betterness might be risk-neutral with respect to well-being. And, happy days of life might not contribute less to well-being the more happy days the agent already has (and similarly for unhappy days). Given that individuals might live arbitrarily long at a constant positive well-being level, this view implies that both well-being and utilities are unbounded. This leads to a prudential analogue of Probability Fanaticism:

Prudential Fanaticism:

i *Positive Prudential Fanaticism* For any probability $p > 0$, and for any finite individual utility u, there is some large enough in-

Monton (2019, §5.7) and Beckstead & Thomas (2020, p. 12).

[31] Broome (1991, p. 217).

dividual utility U such that probability p of U (and otherwise nothing) is prudentially better than the certainty of u for some individual S.

ii *Negative Prudential Fanaticism* For any probability $p > 0$, and for any finite negative individual utility $-u$, there is some large enough negative individual utility $-U$ such that probability p of $-U$ (and otherwise nothing) is prudentially worse than the certainty of $-u$ for some individual S.

To summarize, social utilities might be bounded if the total quantity of well-being is itself necessarily bounded. However, this is not true; therefore, Bounded Expected Totalism requires a social transformation function that takes the total quantity of well-being as input and outputs social utilities. To recap, this social transformation function must be non-linear for three reasons: First, in expectation, an infinite number of individuals might exist, so the total quantity of well-being might be infinite in expectation. But Bounded Expected Totalism requires expected social utilities to be bounded. Secondly, arbitrarily many individuals might exist. In that case, the social transformation function must be non-linear or Bounded Expected Totalism does not avoid Probability Fanaticism. Lastly, even if there is an upper limit to how many individuals might exist, the number of individuals might still be very large. In that case, the social transformation function must be non-linear or Bounded Expected Totalism prescribes fanatical choices in cases that involve tiny probabilities of huge outcomes.

The social transformation function uses the 'total quantity of well-being' as input. To make sense of this notion, well-being must have a cardinal structure. This structure could be primitive, that is, given independently of individual betterness relation on gambles. Alternatively, it could be defined using Bernoulli's hypothesis. If the cardinal structure is defined using Bernoulli's hypothesis, then individual betterness is risk-neutral. But if it is primitive, or defined in some other way, then it is at least initially an open question whether individual betterness is risk-neutral, risk-averse, or what, with respect to well-being. Next, I will show that Bounded Expected Totalism violates Ex Ante Pareto if individual betterness is risk-neutral with respect to well-being. §4 shows that Bounded Expected Totalism violates Ex Ante Pareto if individual betterness is risk-averse with respect to well-being.

3 The risk-neutral case

This section shows that Bounded Expected Totalism violates Ex Ante Pareto if individual betterness is risk-neutral with respect to well-being.

Let well-being levels be represented by real numbers. As argued above, the social transformation function f must be strictly concave on some subset of its domain. For the sake of argument, let's suppose it is strictly concave at 1. Then, there must be some positive constants δ and ϵ such that $f(1) - f(1-\delta) > f(1+\delta+\epsilon) - f(1)$. This is because the smaller benefit (δ) contributes more when added to a population at a lower well-being level than the greater benefit ($\delta + \epsilon$) when added to a population at a higher well-being level.

Next, consider the following prospects:

The Risk-Neutral Case:

Risky Gives a 0.5 probability of a well-being level of $1 + \delta + \epsilon$; otherwise, it gives a well-being level of $1 - \delta$.

Safe Surely gives a well-being level of 1.

Suppose that the betterness relation of some agent, Alice, is risk-neutral with respect to her well-being. Then, Risky is better than Safe for Alice, as Risky gives a higher expectation of well-being than Safe does.

But is Risky also better than Safe impersonally? The answer is no. Given that the constants δ and ϵ are such that $f(1) - f(1 - \delta) > f(1 + \delta + \epsilon) - f(1)$, Safe is impersonally better than Risky (even though Risky is still better than Safe for Alice, given that it gives a higher expectation of her well-being for all positive values of δ and ϵ). The situation is illustrated by the following graph:[32]

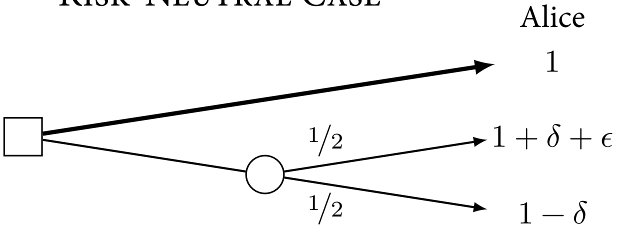
[32] Gustafsson (2022) presents this case to illustrate that *Ex-Post* Prioritarianism violates Ex Ante Pareto, a fact that goes back at least to Rabinowicz (2002). For an overview of this topic, see for example Fleurbaey (2018). See also Broome (1991, Ch. 9). Bounded Expected Totalism coincides with *Ex-Post* Prioritarianism in one-person cases. So, we can appeal to the standard fact that *Ex-Post* Prioritarianism violates Ex Ante Pareto. *Ex-Post* Prioritarianism states the following:

> ***Ex-Post* Prioritarianism:** For all prospects X and Y, $X \succsim Y$ if and only if $\mathrm{EU}_{\mathrm{Soc}}(X) \geq \mathrm{EU}_{\mathrm{Soc}}(Y)$, where
>
> $$\mathrm{EU}_{\mathrm{Soc}}(X) = \sum_{i=1}^{n} p(E_i) \left(\sum_{j=1}^{m} f\left(w(S_{ij})\right) \right).$$

Ex-Post Bounded Expected Totalism differs from *Ex-Post* Prioritarianism because it first sums up everyone's well-being and then converts this sum into social utilities. In contrast, the latter view first converts individuals' well-being levels and then sums up the converted well-being levels. *Ex-Post* Bounded Expected Totalism applies the transformation function to the total quantity of well-being; *Ex-Post* Prioritarianism applies it to the well-being of individuals. On *Ex-Post* Prioritarianism, so-

A Violation of Ex Ante Pareto:

Risk-Neutral Case



Here, the square represents a choice node, while the circle represents a chance node. Going up at the choice node means accepting Safe, and going down at the choice node means accepting Risky. Thus, if we go up, Alice gets a well-being level of 1. On the other hand, if we go down, there are two possible states of the world, each with a 0.5 probability. In state 1, Alice gets a well-being level of $1 + \delta + \epsilon$. And, in state 2, Alice gets a well-being level of $1 - \delta$.

The expected social utility of going up is $\text{EU}_{\text{Soc}}(\text{Safe}) = f(1)$. And, the expected social utility of going down is $\text{EU}_{\text{Soc}}(\text{Risky}) = \frac{1}{2} \cdot f(1+\delta+\epsilon) + \frac{1}{2} \cdot f(1-\delta)$. Given that $f(1) - f(1-\delta) > f(1+\delta+\epsilon) - f(1)$, $\text{EU}_{\text{Soc}}(\text{Risky})$ is less than $\text{EU}_{\text{Soc}}(\text{Safe})$.[33] Thus, going up is impersonally better than going down, according to Bounded Expected Totalism. However, going down is better than going up for Alice (and equally good for everybody else). Thus, we have a violation of Ex Ante

cial utilities are unbounded because the sum of converted well-being levels can be arbitrarily high, given that arbitrarily many individuals might exist. On *Ex-Post* Bounded Expected Totalism, social utilities are bounded because, although the sum of everyone's well-being can be arbitrarily high, the total quantity of well-being has diminishing marginal utility. Similarly, *Ex-Ante* Bounded Expected Totalism differs from *Ex-Ante* Prioritarianism because the former applies the transformation function to the total expected well-being of a prospect, while the latter applies it to the expected well-being of individuals.

[33] By rearranging $f(1) - f(1-\delta) > f(1+\delta+\epsilon) - f(1)$, we get $f(1) + f(1) > f(1+\delta+\epsilon) + f(1-\delta)$. Next, by dividing both sides by 2, we get $f(1) > \frac{1}{2} \cdot f(1+\delta+\epsilon) + \frac{1}{2} \cdot f(1-\delta)$.

Pareto.[34]

To summarize, Bounded Expected Totalism violates Ex Ante Pareto if individual betterness is risk-neutral with respect to well-being. This happens because the social transformation function is concave on some subset of its domain.[35] Consequently, Bounded Expected Totalism is at least sometimes risk-averse with respect to (positive) well-being.

4 The risk-averse case

This section shows that Bounded Expected Totalism violates Ex Ante Pareto even if individual betterness is risk-averse with respect to well-being.[36]

[34] If individual utilities are unbounded above while social utilities are bounded above, then Bounded Expected Totalism violates Ex Ante Pareto in the following case as well:

> **Unbounded individual utilities:**
>
> *Risky** Gives a tiny probability p of a very high positive well-being level w_1 (and otherwise nothing).
>
> *Safe** Surely gives a modest positive well-being level w_2.

Suppose individuals maximize unbounded expected utility, but social utilities are bounded. Then, with some values of p, w_1 and w_2, Risky* is better than Safe* for individuals, but Safe* is impersonally better than Risky*. This is a violation of Ex Ante Pareto. This happens because, in the impersonal case, the additional well-being in w_1 is insufficient to compensate for the tiny probability of obtaining it; however, for individual agents, it is sufficient. Bounded Expected Totalism violates Ex Ante Pareto in a similar case (changing what needs to be changed) if individual utilities are unbounded below while social utilities are bounded below.

[35] The same argument can be applied, changing what needs to be changed, as long as the social transformation function is concave on some subset of its domain—it need not be concave specifically at 1.

[36] It is already known that individual risk attitudes incompatible with Expected Utility Theory can cause tensions with Ex Ante Pareto. See for example Nebel (2020) and Mongin & Pivato (2015). However, the violation of Ex Ante Pareto discussed in this section happens even if the risk aversion is of the kind that is compatible with Expected Utility Theory.

If individual betterness is risk-averse with respect to well-being, then it may no longer be true that Risky is better than Safe for Alice. So, Bounded Expected Totalism might not violate Ex Ante Pareto in the way discussed earlier. If Alice's transformation function corresponds to the social transformation function when Alice is the only person who exists, then Risky is at least as good as Safe for Alice if and only if Risky is at least as good as Safe impersonally (and vice versa). So, Bounded Expected Totalism avoids violating Ex Ante Pareto in the earlier case.

However, how much Alice's well-being contributes to social utility depends on how many individuals exist and what their well-being levels are. The greater the total quantity of well-being, the smaller the contribution of additional well-being is. Suppose that, when Alice is the only person who exists, Alice's loss of δ would reduce social utility by x units, and her gain of $\delta + \epsilon$ would increase it by more than x units. Then, in the one-person case, Risky is better than Safe (both impersonally and, by Ex Ante Pareto, for Alice).[37]

Now change the case; suppose that, besides Alice, there is a large number N of other, unaffected people.

> **Alice and Others:** A large number N of other people have very good lives in state 1 ($p = 0.5$) and neutral lives in state 2 ($p = 0.5$).

[37] Note that this step requires the following version of Ex Ante Pareto:

> **Weak Ex Ante Pareto:** For all prospects X and Y, if X is at least as good as Y for everyone, then X is at least as good as Y.

Also, this step assumes Completeness. Without Completeness, Weak Ex Ante Pareto does not entail that Risky must be better than Safe for Alice if Risky is better than Safe impersonally—they could be incomparable for her.

Risky Gives Alice a well-being level of $1 + \delta + \epsilon$ in state 1 and a well-being level of $1 - \delta$ in state 2.

Safe Gives Alice a well-being level of 1 in states 1 and 2.

In the state where Alice would lose δ (state 2), the other people have neutral lives (i.e., lives whose addition does not increase or decrease the total quantity of well-being). It follows that, no matter how large N is, her loss of δ would still reduce social utility in that state by x units. On the other hand, in the state where Alice would win $\delta + \epsilon$ (state 1), the N people have very good lives. Let α denote the total quantity of well-being of the N people with very good lives. As we increase N, the social utility in state 1 approaches the upper limit of utilities until it comes within x units of the upper limit. Then, increasing Alice's well-being by $\delta + \epsilon$ contributes less than x to social utility in that state. So, the $\delta + \epsilon$ increase in Alice's well-being in state 1 is no longer sufficient to compensate for the possible loss of δ well-being (and x units of utility) in state 2. It follows that, with a sufficiently large N, Safe is impersonally better than Risky. This contradicts Ex Ante Pareto since Risky is better than Safe for Alice, and Safe and Risky are equally good for each of the N additional people.

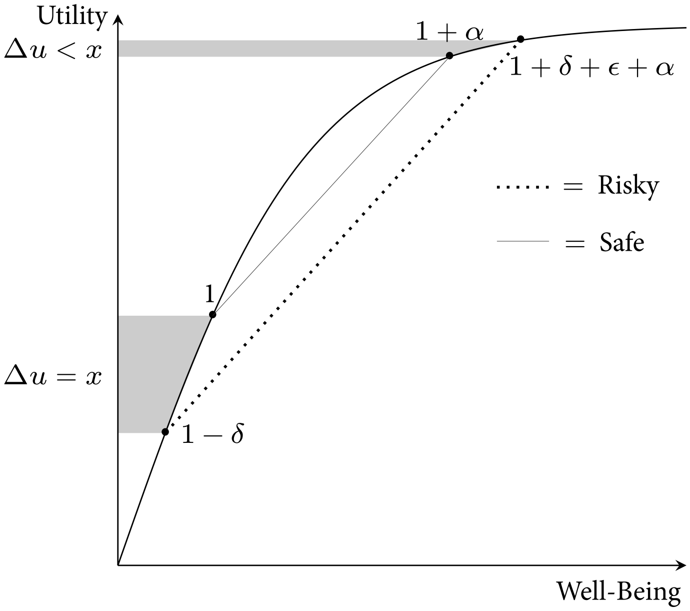
Table 1

Alice and Others

	State 1	**State 2**
p	0.5	0.5
Risky	Alice: $1 + \delta + \epsilon$ Others: α	Alice: $1 - \delta$ Others: 0
Safe	Alice: 1 Others: α	Alice: 1 Others: 0

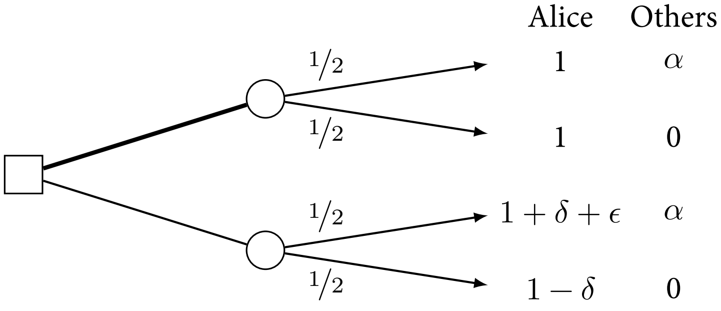
As can be seen from the graph below, Alice's loss of δ (with Risky) reduces social utility by x units, from $u(1)$ to $u(1 - \delta)$. However, her gain of $\delta + \epsilon$ (with Risky) increases social utility by less than x units, from $u(1 + \alpha)$ to $u(1 + \delta + \epsilon + \alpha)$. Thus, Safe is impersonally better than Risky. But we have assumed that Alice's gain of $\delta + \epsilon$ increases her own utility by more than x units. So, Risky is better than Safe for Alice, and we have a violation of Ex Ante Pareto.

Alice and Others



A Violation of Ex Ante Pareto:

Risk-Averse Case



To summarize, Bounded Expected Totalism violates Ex Ante Pareto even if individual betterness is risk-averse with respect to well-being.

5 Bounded above and below

This section gives a general argument for why Bounded Expected Totalism must violate Ex Ante Pareto if social utilities are bounded above and below. This argument shows that a violation of Ex Ante Pareto must happen regardless of whether individual utilities are bounded or unbounded and whether individual betterness is risk-neutral, risk-averse or risk-seeking. But first, to introduce some background, I will discuss a case that shows how Bounded Expected Totalism violates Ex Ante Pareto if individual well-being is unbounded and both individual and social utilities are bounded above and below.

5.1 Unbounded individual well-being

Assuming that overall betterness can be represented with an expectational utility function, social utilities must be bounded above and below in order to avoid both Positive and Negative Probability Fanaticism. Similarly, to avoid Positive and Negative Probability Fanaticism in the prudential case, individual utilities must be bounded above and below. This will lead to a violation of Ex Ante Pareto if individual well-being is unbounded. Consider the following prospects:

Risky vs. Safe**:**

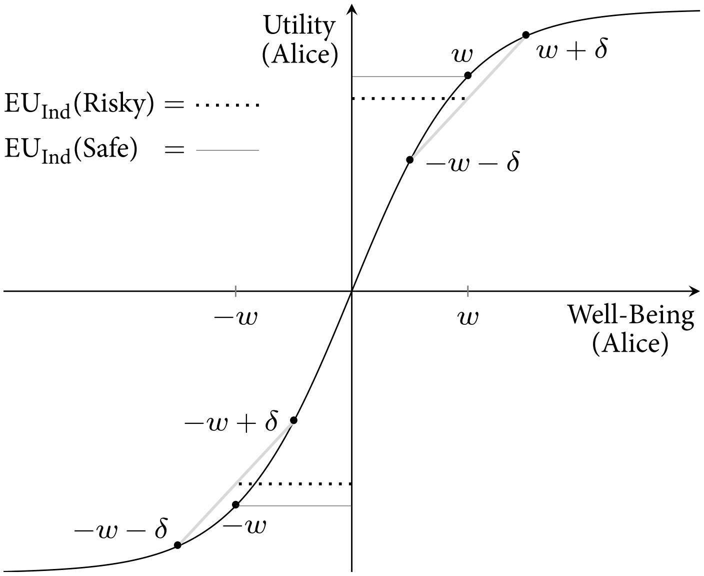
*Risky*** Gives a 0.5 probability of δ additional well-being; otherwise, it decreases well-being by $-\delta$.

*Safe*** Does not increase or decrease well-being.

As explained before, if utilities are bounded above, then (at least at some point) Alice's transformation function is concave with positive well-being; additional well-being matters less the happier Alice already is. This means that, at least sometimes, Alice's betterness relation is risk-averse with respect to her well-being. On the other hand, if utilities are bounded below, then (at least at some point) Alice's transformation function is convex with negative well-being; additional unhappiness matters less the unhappier Alice already is. This means that, at least sometimes, Alice's betterness relation is risk-seeking with respect to her ill-being.

In expectation, neither Risky** nor Safe** affects Alice's well-being. So, which of Risky** and Safe** is better for Alice can depend on whether Alice is overall happy or unhappy (see the graph below). With some positive background well-being level w, Safe** is better than Risky** for Alice. In contrast, with some negative background well-being level $-w$, Risky** is better than Safe** for Alice.

Risky** vs. Safe**



Next, to avoid Positive and Negative Probability Fanaticism, social utilities must also be bounded above and below. If social utilities are bounded above, then (at least at some point) the social transformation function is concave with a positive total quantity of well-being. This means that, at least sometimes, the overall betterness relation is risk-averse with respect to well-being. On the other hand, if social utilities are bounded below, then (at least at some point) the social transformation function is convex with a negative total quantity of well-being. This means that, at least sometimes, the overall betterness relation is risk-seeking with respect to well-being. So, with some positive total quantity of well-being W, Safe** is impersonally better than Risky**. On the other hand, with some negative total quantity of well-being $-W$, Risky** is impersonally better than Safe**.

But now consider the following cases:

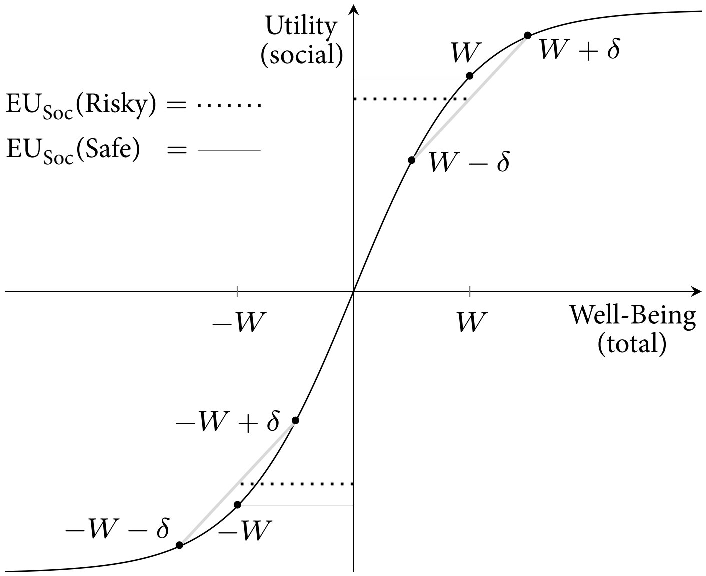
Alice and Others*: Alice's well-being will increase, decrease or stay the same depending on the choice and result of Risky** and Safe** (and nobody else is affected).

Sad Alice in a happy world: Alice has a baseline well-being level of $-w$. The total quantity of well-being in the world is W (includes Alice's well-being).

Happy Alice in a sad world: Alice has a baseline well-being level of w. The total quantity of well-being in the world is $-W$.

Risky** vs. Safe**:

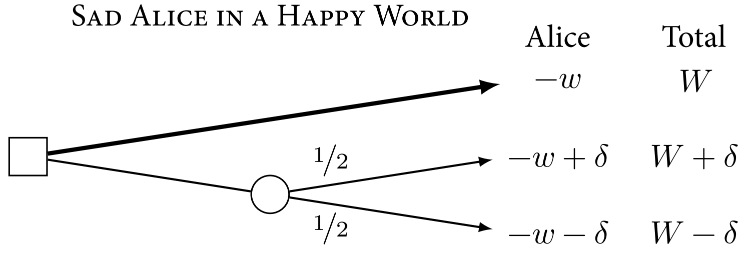
Alice and Others



When Alice's baseline well-being level is $-w$, but the total quantity of well-being in the world is W, Safe** is impersonally better than Risky**, but Risky** is better than Safe** for Alice. On the other hand, when Alice's baseline well-being level is w but the total quantity of well-being in the world is $-W$, Risky** is impersonally better than Safe**, but the reverse is true for Alice. Consequently, Bounded Expected Totalism violates Ex Ante Pareto if both individual and social utilities are bounded above and below but individual well-being is unbounded.

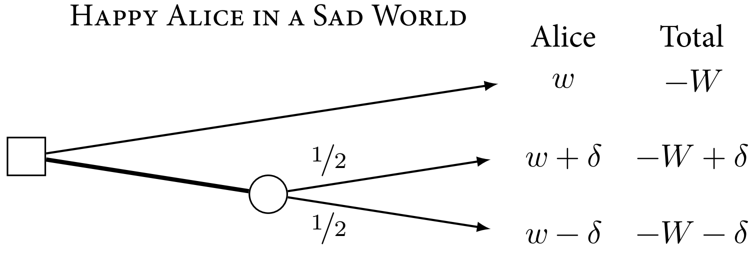
A Violation of Ex Ante Pareto:

Sad Alice in a Happy World



A Violation of Ex Ante Pareto:

Happy Alice in a Sad World



5.2 The general argument

Next, I will give a general argument which shows that Bounded Expected Totalism must violate Ex Ante Pareto, regardless of whether individual utilities are bounded or unbounded and whether individual betterness is risk-averse, risk-neutral or risk-seeking. The general argument goes as follows: Fix any $\delta > 0$. The following two claims are true:

(1) If Risky** is impersonally at least as good as Safe** no matter how much total well-being there is in the background population, then social utility is unbounded above.

(2) If Safe** is impersonally at least as good as Risky** no matter how much total well-being there is in the background population, then social utility is unbounded below.

If social utility is bounded above and below, there must be a counterexample to Ex Ante Pareto. Suppose, for example, that Risky** is at least as good as Safe** for Alice. This could be because Alice's betterness relation is risk-neutral with respect to her well-being and Risky** is therefore equally as good as Safe** for Alice. Alternatively, Alice's betterness relation might be risk-seeking. Either way, if social utilities are bounded above, then (1) shows that Risky** cannot be impersonally at least as good as Safe** no matter how much total well-being there is in the background population. So, with some total quantity of well-being, Safe** is impersonally better than Risky**—which contradicts Ex Ante Pareto.

Similarly, suppose that Safe** is at least as good as Risky** for Alice. Again, this might be because Alice's betterness relation is risk-neutral with respect to her well-being. Alternatively, it could be because her betterness relation is risk-averse. However, given that social utilities are bounded below, (2) shows that Safe** cannot be impersonally at least as good as Risky** no matter how much total well-being there is in the background population. So, with some total quantity of well-being, Risky** is impersonally better than Safe**, contrary to Ex Ante Pareto.

Proof of (1) goes as follows: Consider background populations with total well-being levels of 0, δ, 2δ, 3δ, and so on. Let $x = f(\delta) - f(0)$. If Risky** is impersonally at least as good as Safe** with respect to all these background populations, then the difference between $f(n\delta)$ and $f((n-1)\delta)$ is at least as great as the difference between $f((n-1)\delta)$ and $f((n-2)\delta)$, for each $n > 2$. It follows that $f(n\delta)$ is at least as great as nx. Thus, f is unbounded above. One can give a similar proof for (2). So, if social utilities are bounded above and below, there must be a counterexample to Ex Ante Pareto, regardless of whether individual utilities are bounded or unbounded and whether individual betterness is risk-averse, risk-neutral or risk-seeking.

To summarize, this section first showed that Bounded Expected Totalism violates Ex Ante Pareto if both individual and social utilities are bounded above and below. Next, this section presented a general proof to the effect that Bounded Expected Totalism must violate Ex Ante Pareto regardless of whether individual betterness is risk-neutral, risk-averse or risk-seeking and whether individual utilities are bounded or unbounded.

6 Harsanyi's social aggregation theorem

This section discusses how the earlier examples relate to a famous result in this area, namely, Harsanyi's social aggregation theorem.

Harsanyi's social aggregation theorem shows that if both individual and social betterness relations can be given an expected utility representation, and the overall betterness relation satisfies Ex Ante Pareto, then social utilities are weighted sums of individual utilities.[38] Let me explain Harsanyi's premises in more detail. The first premise says that each individual's betterness relation obeys the von Neumann-Morgenstern axioms.[39] So, the individual betterness relation can be represented by an expectational utility function. The second premise says that the overall betterness relation obeys the von Neumann-Morgenstern axioms. So, overall betterness can also be represented by an expectational utility function. The third premise is Ex Ante Pareto.[40] The conclusion of Harsanyi's theorem is that social utilities are

[38] Harsanyi (1955). Harsanyi (1955) uses individual utilities to describe individual preferences. But we may reinterpret them as describing individual betterness instead of individual preferences. See Broome (1991).

[39] Harsanyi (1955) uses Marschak's (1950) versions of the von Neumann & Morgenstern (1947) axioms. Marschak's (1950, p. 117) Postulate II states:

> **Postulate II (Continuity):** If $X \succ Y \succ Z$, then there is a probability $p \in (0, 1)$ such that $Y \sim XpZ$.

This postulate implies, in a similar way as shown before, that utilities must be bounded.

[40] Harsanyi (1955) uses Pareto Indifference in the original formulation of the theorem, while Harsanyi (1977, p. 65) uses Weak Ex Ante Pareto:

> **Pareto Indifference:** For all prospects X and Y, if X and Y are equally good for everyone, then X and Y are overall equally good.

> **Weak Ex Ante Pareto:** For all prospects X and Y, if X is at least as good as Y for everyone, then X is overall at least as good as Y.

weighted sums of individual utilities. Thus, overall betterness can be represented as maximizing the expectation of a weighted sum of individual utilities. If, in addition, we assume equal weighting for all individuals, then this theorem shows that the social utility function must be a sum of individual utilities.[41]

Harsanyi's theorem shows, in other words, that if individual and overall betterness relations are represented by expectational utility functions, then in order to satisfy Ex Ante Pareto, the social utility function must be a linear combination of individual utilities. Earlier in this chapter, I showed that Total Utilitarianism combined with Bounded Expected Utility Theory violates Ex Ante Pareto.[42] There-

Using Weak Ex Ante Pareto instead of Pareto Indifference guarantees that positive individual well-being contributes *non-negatively* to social utilities. Using Ex Ante Pareto instead of Weak Ex Ante Pareto guarantees that positive individual well-being contributes *positively* to social utilities. See Weymark (1994) on Harsanyi's theorem with different Pareto principles.

[41] Broome (1991, §10) argues that Harsanyi's social aggregation theorem, together with Bernoulli's hypothesis, leads to utilitarianism.

[42] As mentioned in footnote 40, the original argument by Harsanyi (1955) uses Pareto Indifference instead of Ex Ante Pareto. Bounded Expected Totalism also violates this condition if individual betterness satisfies the von Neumann-Morgenstern axioms. Consider the following prospects:

> **Alice and Bob:**
>
> *Risky* Gives Alice and Bob a well-being level of 1 in state 1 ($p = 0.5$) and a well-being level of 0 in state 2 ($p = 0.5$).
>
> *Safe* In state 1 ($p = 0.5$), Alice gets a well-being level of 1 and Bob a well-being level of 0. In state 2 ($p = 0.5$), Alice gets a well-being level of 0 and Bob a well-being level of 1.

Risky and Safe are equally good for both Alice and Bob. So, by Pareto Indifference, Risky and Safe are impersonally equally good. Next, recall that the social transformation function must be strictly concave on some subset of its domain if social utilities are bounded above (for the reasons discussed in §2.1). We may suppose it is strictly concave on the interval [0, 2]. Consequently, Safe is impersonally better than Risky. This violation of Pareto Indifference happens because when the social transformation function is strictly concave, it is impersonally better to spread the total quantity of well-being across different states than to have it all in one state. But individual betterness is indifferent to how the well-being of different individuals is spread across states.

fore, if one accepts Bounded Expected Totalism, that premise of Harsanyi's theorem fails. The reason that led to its failure was that a non-linear social transformation function is needed because the number of individuals might be infinite or arbitrarily large. In fact, it is unsurprising that one of Harsanyi's premises must be rejected; if the number of individuals might be infinite or arbitrarily large, then social utilities cannot be weighted sums of individual utilities because this could lead to unbounded social utilities.[43,44] So, given that a bounded expected totalist rejects Harsanyi's conclusion, they cannot accept all his premises.

This is worrying because Harsanyi's theorem is often considered one of the best arguments for utilitarianism. The conclusion of Harsanyi's theorem is that, for any fixed and finite population, social utility is an affine (or linear) function of total individual utility. However, once we consider the possibility of an infinite or arbitrarily large population, we find that social utility must be non-linear if social utilities are bounded and additive with individual utilities.[45] And this leads to violations of Ex Ante Pareto.

All this can be taken to support *Average Utilitarianism*, namely, the view that one population is better than another if and only if the average well-being it contains is greater.[46] Alternatively, these cases might be taken to undermine Bounded-

[43] See Blackorby et al. (2007) for an extension of Harsanyi's social aggregation theorem to variable populations.

[44] As mentioned earlier, this need not be true. See footnote 18 on p. 70.

[45] Harsanyi (1977, p. 60) himself discusses what he calls the 'boundary problem for the society', namely, whose utility functions ought to be included in our social-welfare function. He considers whether to include, for example, higher animals, distant future generations, robots or the inhabitants of other planets. However, he does not mention the possibility that doing so might lead to infinite or arbitrarily large populations.

[46] Average Utilitarianism does not require a non-linear social transformation function; if indi-

ness (and Continuity). One might accept, for example, *Unbounded Expected Totalism*, namely, the view that combines Total Utilitarianism and Expected Utility Theory with an unbounded utility function. However, this view cannot be supported by a version of Harsanyi's theorem that relies on the von Neumann-Morgenstern axiomatization of Expected Utility Theory, as this axiomatization has Continuity as one of its axioms. But one might attempt to justify Unbounded Expected Totalism with a Harsanyi-style argument that does not rely on Continuity.[47] Finally, as mentioned earlier, the arguments in this chapter might be taken to support Probability Discounting indirectly. As I will explain in Chapter 3 of this thesis, Probability Discounting also leads to violations of Ex Ante Pareto.[48] But given that both theories violate Ex Ante Pareto, the plausibility of Ex Ante Pareto does not favor Bounded Expected Totalism over Probability Discounting.

7 Conclusion

This chapter has shown that Bounded Expected Totalism violates Ex Ante Pareto. Separate examples of Ex Ante Pareto violations were given for risk-neutrality and

vidual utilities are bounded, then the average of those must also be bounded. So, Average Utilitarianism avoids violating Ex Ante Pareto. However, Average Utilitarianism has other implausible implications, such as the *Sadistic Conclusion* (Arrhenius 2000, p. 251):

> **The Sadistic Conclusion:** When adding people without affecting the original people's welfare, it can be better to add people with negative well-being rather than positive well-being.

[47] Fleurbaey (2009) gives such an argument using statewise dominance and anonymity instead of the von Neumann-Morgenstern axioms. Relatedly, McCarthy et al. (2020) show that one can argue for Expected Utility Theory with an unbounded utility function from Pareto and anonymity.

[48] See also Kosonen (2021).

risk-aversion. A general argument to the effect that Bounded Expected Totalism must violate Ex Ante Pareto was also given. Lastly, the implications of these cases for Harsanyi's social aggregation theorem were discussed.

The violations of Ex Ante Pareto happen because there is a non-zero probability that an infinite or arbitrarily large number of individuals exist. These Ex Ante Pareto violations also happen if one wishes to avoid Probability Fanaticism. Since Bounded Expected Totalism cannot avoid Probability Fanaticism without violating Ex Ante Pareto, these violations of Ex Ante Pareto undermine the plausibility of Bounded Expected Totalism as an alternative to Probability Fanaticism.

To conclude, combining Total Utilitarianism and Expected Utility Theory with a bounded utility function results in violations of Ex Ante Pareto: The combination of these views implies that a prospect can be impersonally better than another prospect even though it is worse for everyone who is affected by the choice.

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